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SHIP HULL VIBRATIONS - 5  
ANALYSIS OF HULL STRUCTURES AS  
APPLIED TO SSB(N) 598 GEORGE WASHINGTON

by

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## ABSTRACT

A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SSB(N) 598 GEORGE WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stern of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.

## SUMMARY

### STATEMENT OF THE PROBLEM

A previous prediction<sup>2</sup> of the amplitude of longitudinal and vertical bending vibration excited by the propeller for the submarine GEORGE WASHINGTON, SSB(N) 598, gave results that agreed moderately well with tests in longitudinal vibration and did not agree for vertical bending vibration. This study has been undertaken to explore the factors that influence the structural response of the hull and eventually (in a subsequent report) to see whether it is possible to make a more accurate prediction of the amplitudes of propeller excited hull vibration.

### WORK SUMMARY

1. A computing machine program that is sufficiently flexible to deal with parallel sub-systems (e. g. , the propulsion system), sprung masses and concentrated and hysteresis damping in axial and bending vibration has been developed.
2. The mass-elastic characteristics of the SSB(N) 598 GEORGE WASHINGTON have been determined in considerable detail.
3. The effects of various approximations such as omitting the propulsion sub-system, treating sprung masses as rigidly connected, neglecting rotary inertia, representing the hull with a limited number of mass points, spacing the mass points uniformly rather than to conform to hull mass and stiffness variations, have been investigated
4. The effects of various amounts of hysteresis damping in the hull have been investigated.

## CONCLUSIONS

1. In predicting the vibration response of a ship in axial and bending modes, it is necessary to include the propulsion system (propellers and shafting) as an elastic sub-system in the calculations if reliable results are to be obtained.
2. Hull damping not only reduces the vibration amplitude in the vicinity of the excitation but also attenuates the relative amplitude at locations remote from the point of excitation.
3. In order to obtain reliable predictions of vibration response of a ship it is necessary to define the hull mass and stiffnesses accurately. A less accurate definition is required if only hull frequencies are desired. Because of the attenuation caused by damping, the accurate definition of the hull is particularly important in the regions near the excitation.
4. On the submarines the effect of including rotary inertia of the hull cross sections upon the response of the ship are so small as to be negligible.
5. When a careful effort has been expended to define the hull mass and stiffnesses accurately, it is not advisable to use only a few mass points to represent the hull. The computation program can accept many points at a negligible extra cost. The greater number of points will result in better accuracy and a better definition of the hull motions will be obtained.
6. The hull response is sensitive to the stiffness of the connections between the propulsion system and the hull, and therefore careful attention should be paid to these stiffnesses. By controlling these stiffnesses to locate the resonant frequencies between the propulsion sub-system and the hull, it is possible to influence the hull vibration amplitudes at low speeds as compared with those at high speeds.
7. Although the total weight of the sprung masses is small compared to the weight of the hull, including them as sprung weights rather than as solidly mounted weights has an

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important effect upon the hull response.

## RECOMMENDATIONS

If reliable estimates of the response of a ship hull to vibratory excitation are to be obtained, it is necessary to determine the mass and elastic characteristics of the hull in considerable detail. Particular attention should be paid to the propulsion sub-system and its connection to the hull. The local springing of hull weights must be included.

## I INTRODUCTION

In an early report<sup>1</sup> CONESCO outlined a program for predicting the amplitude of propeller excited hull vibration and presented ways of estimating the excitation arising from the propellers working in irregular wakes. A subsequent report<sup>2</sup> applied this program to a prediction of the longitudinal and vertical bending propeller excited vibrations on the USS GEORGE WASHINGTON, SSB(N) 598. For this purpose the excitation was computed by the methods of the earlier report but the structural response of the hull was determined from existing calculations<sup>3</sup>. It was considered that if the ship resonant frequencies as measured on trials and the amplitudes of vibration on the flanks of the resonances agreed with those that were predicted, that the method of prediction would be validated. The amplitude of vibration at the resonances would depend upon more detailed considerations of damping in the calculations and more experimental comparison between the damped calculations and trial results than could be made at the time.

When this comparison was made it was found that there was a moderate support for the calculation method as applied of longitudinal vibration of the hull since the predicted amplitudes of vibration on the flanks agree moderately well with measured values and the calculated and measured natural frequencies agreed within about 10%. For the bending calculation there appeared to be no agreement. However, the representation of the hull in the axial response calculations took account of the elasticity of the connection between the propeller and the hull where the bending calculation did not.

In an effort to improve the accuracy of the computation process and to study the effects of damping on the hull vibration amplitude, a thorough study of the structural response of the hull has been undertaken. To accomplish this it has been necessary to develop a new program for computing hull response in bending which is sufficiently adaptable to include not only the mass and elasticity of the elements in the propulsion system, but also the elastically supported elements throughout the ship. A procedure for computing the response of a ship that is represented in the most general way of a beam having

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distributed mass and stiffness was also considered but because of complexity in the programming could not be completed and is therefore not reported at this time.

The results of these studies, involving (1) a more precise representation of the hull mass and structure (2) the inclusion of elastic sub-systems and (3) an investigation of the effects of simplifications and assumptions made in the calculations, are presented in this unclassified report. Because the actual vibration of the ship falls in the realm of classified information, the comparison of calculated and experimental amplitudes of vibration will be presented in a subsequent classified report.

## II METHOD OF SOLUTION

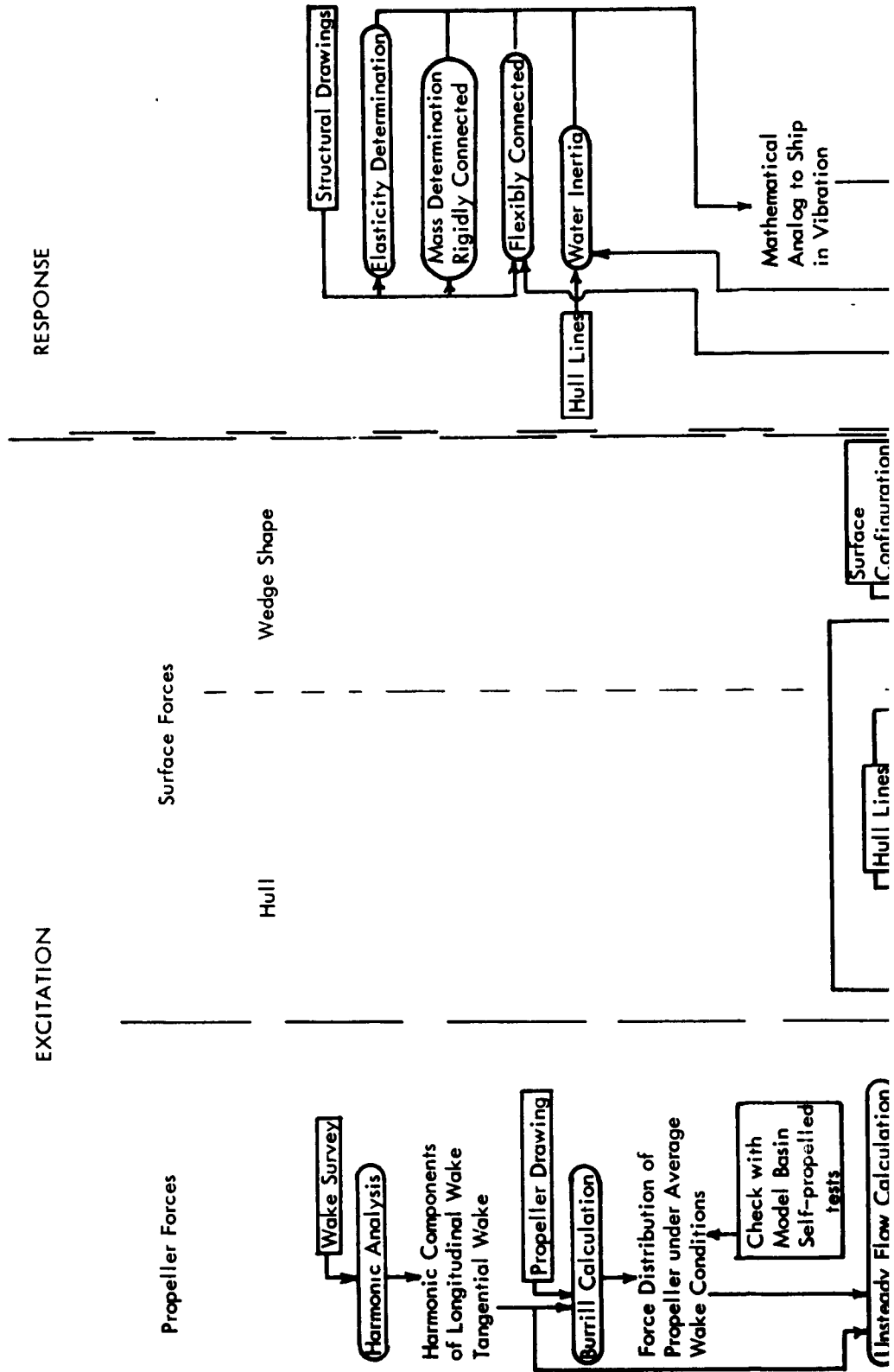
The method of computing the amplitudes of the propeller excited hull vibration falls into two categories: (1) the calculation of the exciting forces and (2) the calculation of the response of the hull to these exciting forces. This is essentially the standard procedure followed in the calculation of the forced vibrations of any system but in the case of ships involves many difficulties. The flow of work in the prediction process is diagrammed in Figure 1. The prediction of the forces has been discussed in previous reports,<sup>2, 4</sup> This report is concerned solely with the response of the hull to these forces.

## III DEFINITION OF THE MASS AND ELASTIC PROPERTIES OF THE SHIP

It should be pointed out that since the ship under consideration is a submarine, the definition of the mass and stiffness can be made quite exact. The requirement of adequate transverse strength results in heavy structural members which carry their loads without buckling or local resonances. Furthermore, the shape is readily adaptable to theoretical predictions of the inertias of entrained water. It should not be assumed however, that even the submarine can be defined precisely. The flexibilities and water inertias associated with the control surfaces are inadequately handled in the present study and there are questions about the stiffness of the superstructure and the mass of the free flooding water

1

# SEQUENCE OF CALCULATIONS FOR ESTIMATING THE AMPLITUDE OF PROPELLER EXCITED HULL VIBRATION



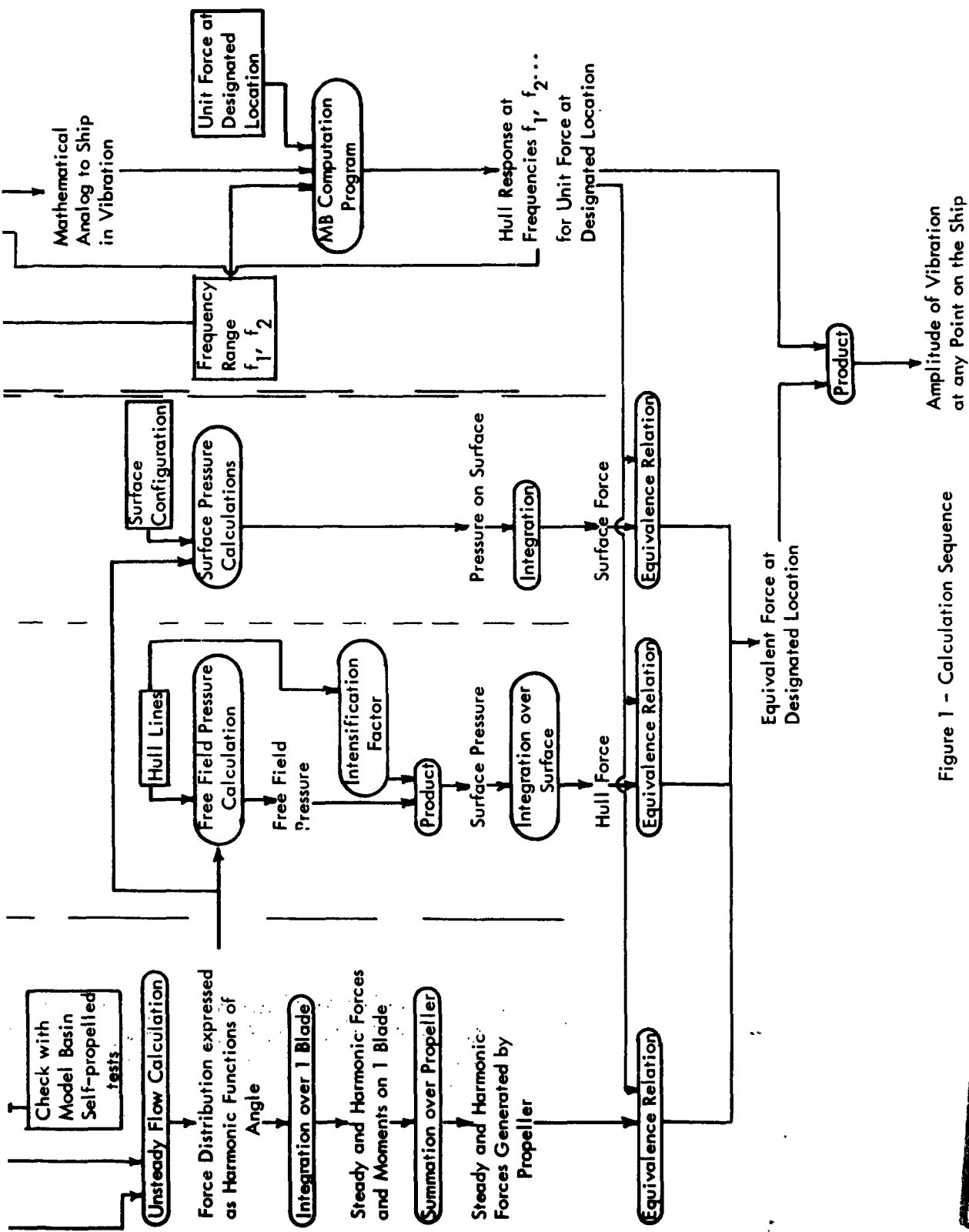


Figure 1 - Calculation Sequence

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between the superstructure and the hull.

In this study the mass distribution and the stiffness distribution were determined to a rather high degree of precision and then plotted. From these plotted results a variety of hull representations were developed which seemed to fit the distributions of mass and stiffness well. A small scale plot of stiffnesses is given in Figure 2 and of weight distribution in Figure 3. The weight moments of inertia of the hull are presented in Figure 4. (Apparently E. B. division feels that the entrained water contributes to moment of inertia. We feel that the contribution, as to the axial mass, is negligible). The weight and stiffness of the propulsion system in longitudinal vibration is given in Figure 5 and in vertical vibration in Figure 6. The characteristics of the sprung mounted weights and their mountings are given in Table I. A discussion of the methods by which these quantities were determined and the factors that entered into their determination are given in Appendices A-H.

#### IV CALCULATION PROCEDURE

For the longitudinal vibration it was found that an existing Taylor Model Basin Code was adequate for handling all of the variables that it was desired to impose upon the problem. For the analysis of the bending, all of the existing codes were quite inadequate. CONESCO originally proposed some additions by which the existing codes might be made workable but the addition of these additional complications to an already inefficient code was not considered desirable. Dr. Cuthill (one of the authors of this report) therefore prepared an entirely new code which is adaptable to a wide variety of hull representations. This code can deal not only with the bending but also is sufficiently general to cover the longitudinal vibration and with small changes can deal with coupled longitudinal and vertical bending or coupled torsion and transverse bending. Because it was developed after the hull had been defined, the present calculations do not fully exploit the potentials of the new code (specifically in the treatment of control surfaces). A presentation of the basis for this code and the way it is used is given in Appendix I. This general bending

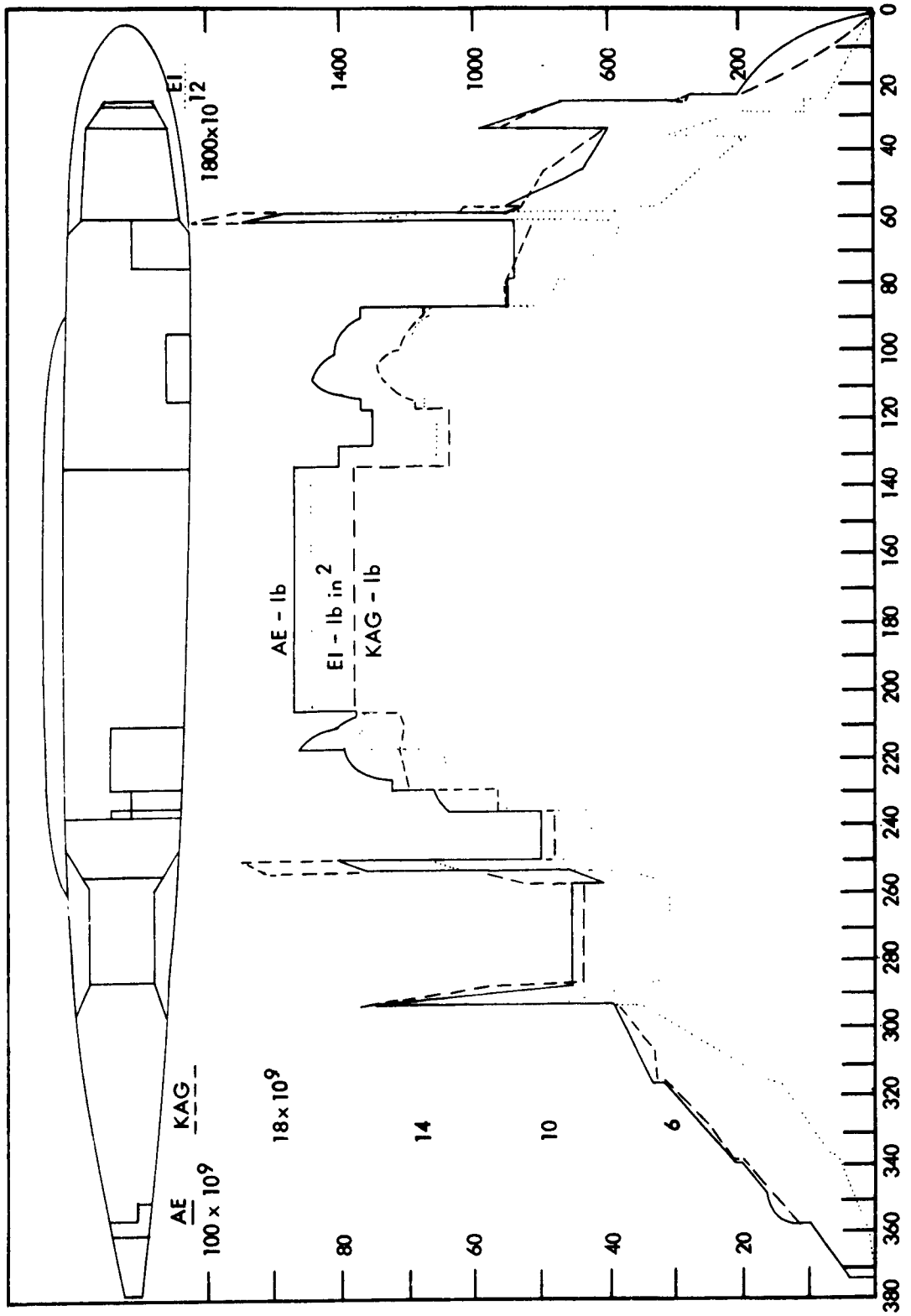
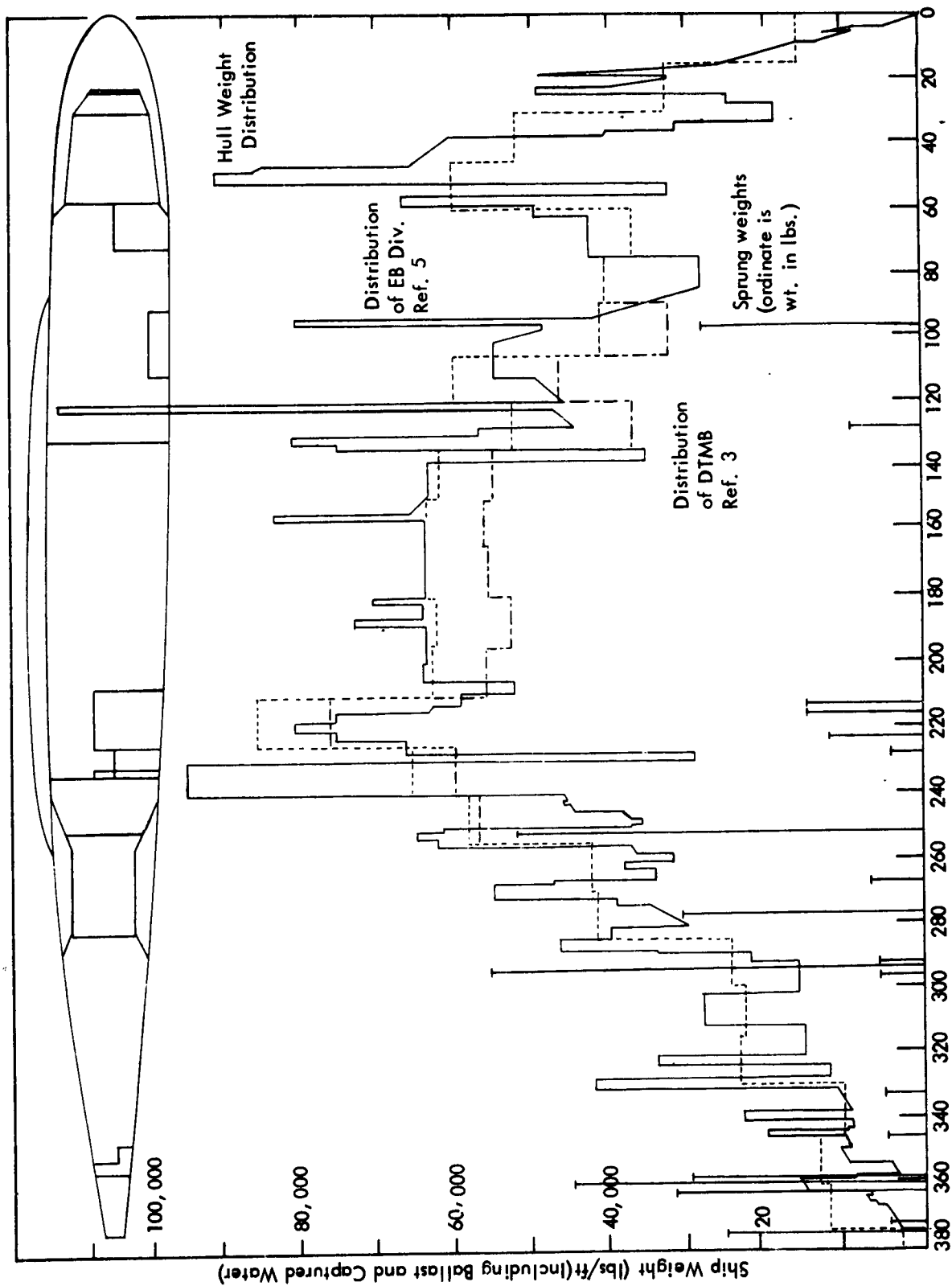


Figure 2 - SSB(N) 598 - Axial, Shear and Bending Stiffness



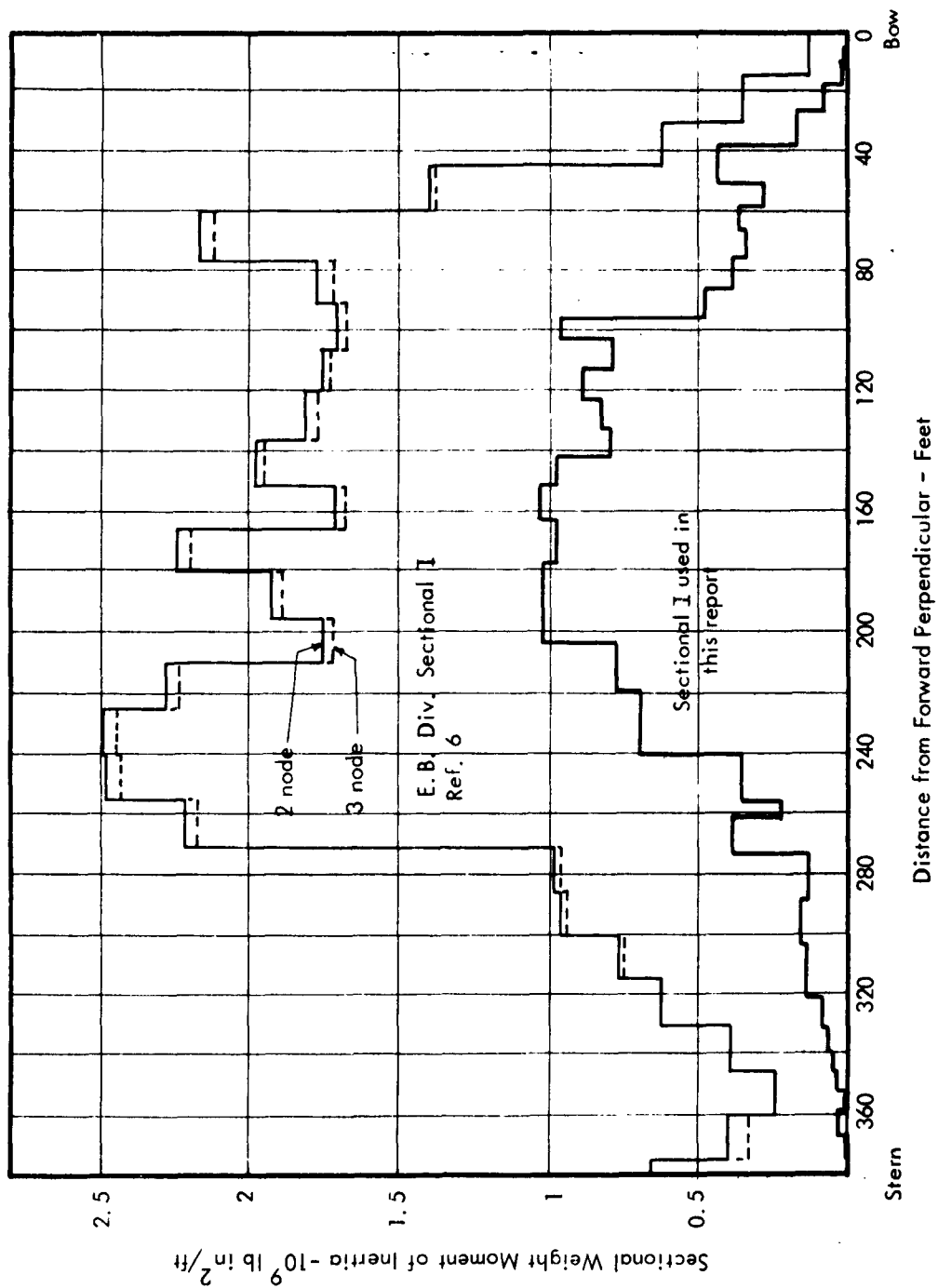
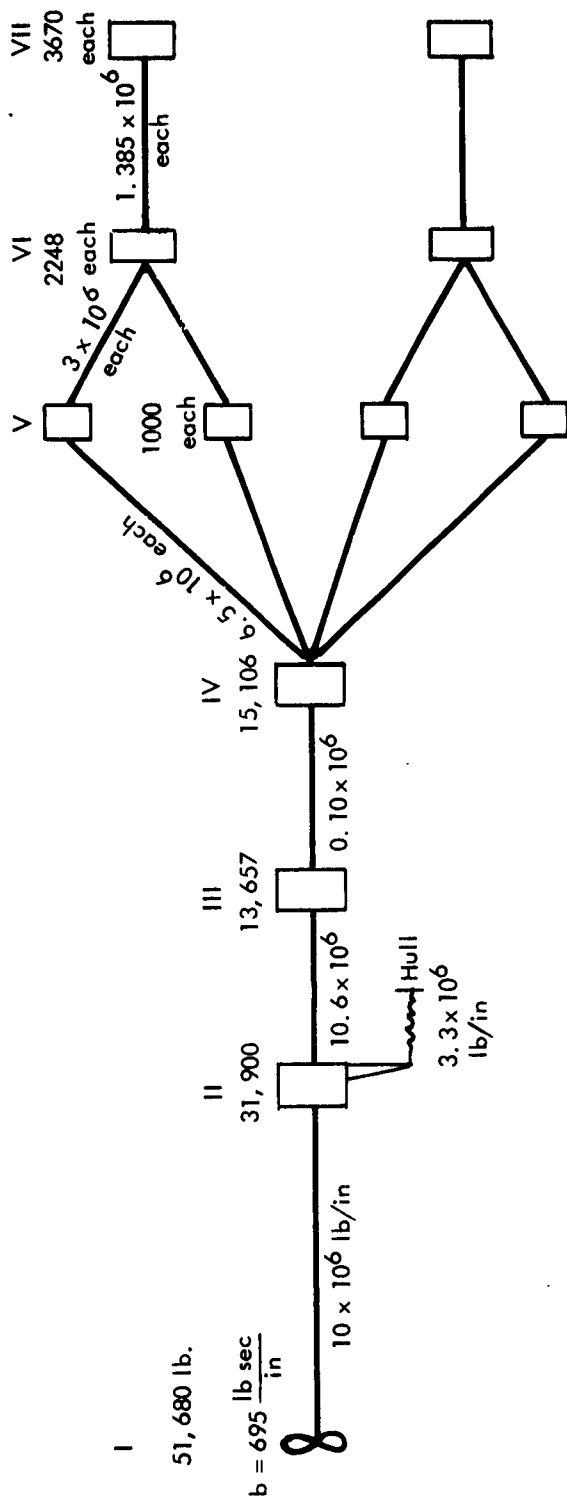


Figure 4 - Sectional Weight Moments of Inertia - SSB(N) 598



Actual Series - Parallel System

Equivalent Series System

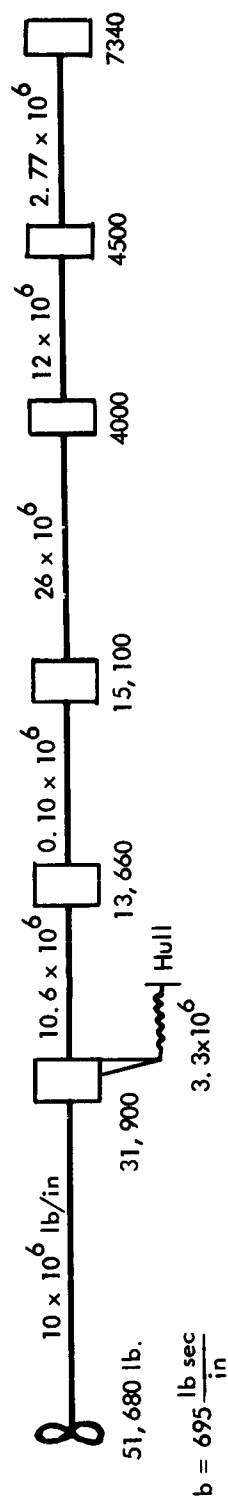


Figure 5-SSB(N) 598 Weight and Stiffness of Propulsion System in Longitudinal Vibration

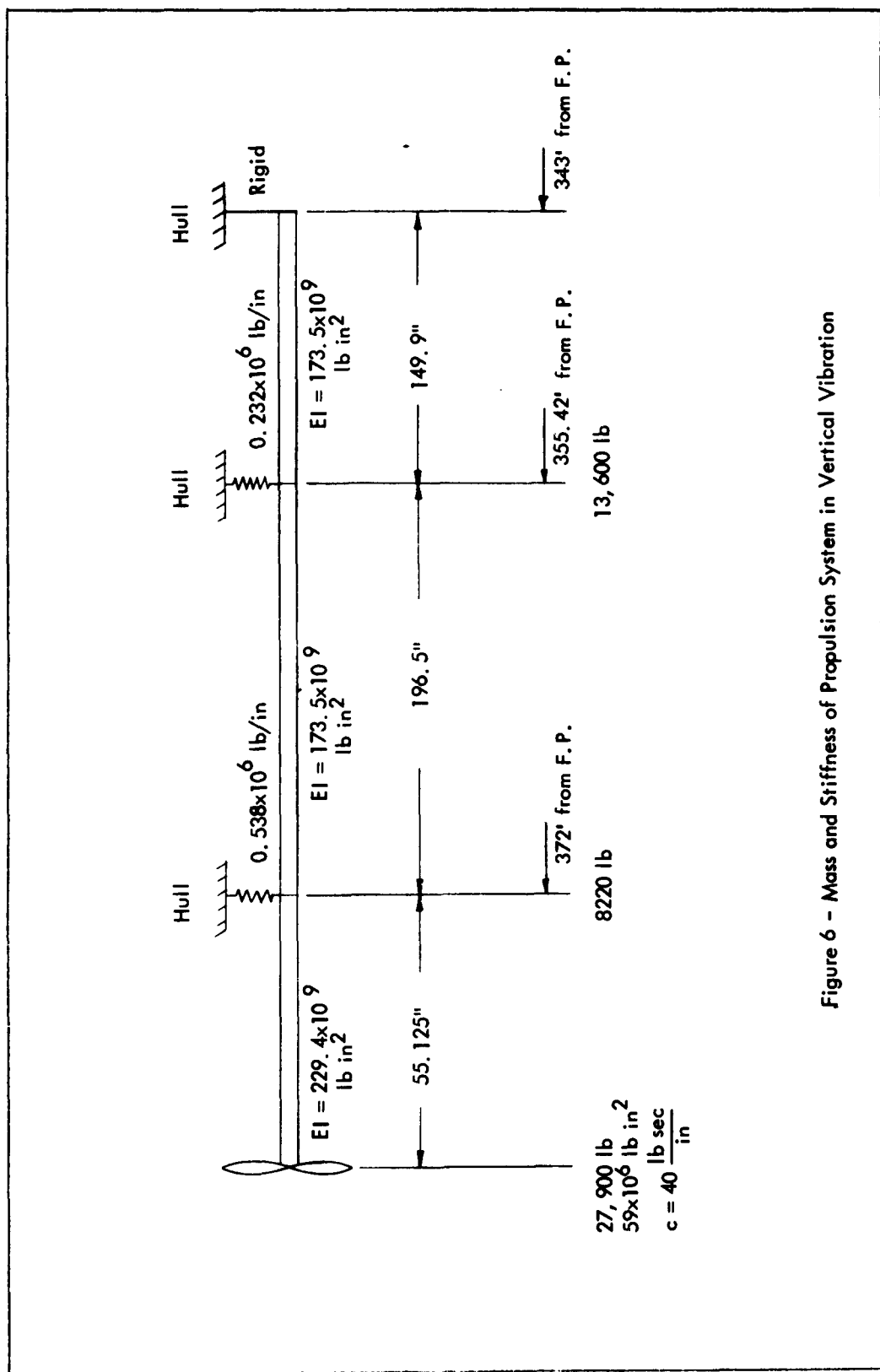


Figure 6 - Mass and Stiffness of Propulsion System in Vertical Vibration

TABLE 1 - SSB(N) 598 GEORGE WASHINGTON SPRUNG WEIGHTS

Flexibly-Mounted Element	Distance from F. P. (feet)	Weight (lbs)	Longitudinal		Vertical	
			Stiffness lb/in	Damping lb sec/in	Stiffness lb/in	Damping lb sec/in
Each of two Fair-water diving planes	98	Long. 12,771 Vert. 7,750 37,300	Rigid		Rigid 122,200	60n
Gyro. Hyd. System	125	6,900	12,400	24.6	31,440	29.3
L. P. Blower	128	3,145	5,720	11.5	13,600	12.6
O2 Gen. Plant	213	15,200	11,600	27.8	50,000	60.7
Air Cond. Set	217	15,200	11,200	19.3	51,600	41.6
400 Cycle M. G. sets	224	12,261	8,100	15.2	38,700	33.3
H. P. Air Comp.	228	3,450	6,160	25.25	14,400	14.5
Main Coolant Pumps	251.5	51,893	307,000	192	Rigid	
Trim and Drain Pumps	268	5,300	9,000	15.9	16,000	16.4
300 KW M. G. Sets	276	32,920	20,640	107	46,000	104
H. P. Air Comp.	292	3,991	7,150	14.2	16,700	16.7
Air Cond. Sets each of 2	294	27,470	36,120	81.5	80,500	79.1
H. P. Air Comp.	298	3,991	7,150	14.2	16,700	16.7
Hydraulic Plant	346	4,300	6,400	13.6	16,800	19.8
Each of 2 stabilizers + Stern Planes	360	Long. 36,495 Vert. 68,100 125,000	Rigid		Rigid 1,820x10 <sup>6</sup>	110n

\* Damping is located between mass and foundation except for fairwater and stern planes which are to ground. Fairwater and stern plane damping constant is proportional to ship speed. This proportionality is represented by  $n$ , the frequency of oscillation, cps, for the case of a 5-bladed propeller.

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response code carries the AML problem number designation 840-277.

## V DAMPING

Damping on a complex structure like a ship is difficult to define – in fact, it can only be determined by experience. However, there are certain types of damping that can be calculated by theory or from the results of experiments external to the ship. Among these are the damping of the propeller in longitudinal, torsional and transverse motion, the aerodynamic damping of control surfaces and the damping in the rubber mounts used under flexibly mounted machinery. Although these are significant sources of damping, the major damping is in the hull. By experiments and theory it is known that this damping is in the hull itself not in the water and is thus related to the slope of the deflection curve, not the absolute amplitude, i. e., the relative amplitudes between adjacent points. In the damping of structures it has been found that one of the most reasonable ways to deal with this type of damping is to tie it with the stress in the structure. If it is assumed that a certain percentage of the stress energy in a volume of stressed material is dissipated because of damping, then it is possible to incorporate the effects of this damping in the calculations by the simple expedient of making the modulus of elasticity a complex term. It should be understood that the actual damping in a ship is much larger than that due to the hysteresis in the material. Most of the damping probably comes from sliding between elements of structure and cargo. It is tied to stress only because this procedure offers a good basis for calculations, and is not incompatible with experience. The actual value of the damping constants can only be determined by many comparisons between carefully calculated amplitudes of vibration and those obtained on trials. The damping constants will probably vary from ship to ship. It will be noted in the subsequent results that the damping not only reduces the amplitudes of vibration in the vicinity of the exciting force but also reduces the transmission of the vibration into parts of the structure remote from the excitation.

## VI REPRESENTATION OF THE HULL IN THE CALCULATIONS

Because the hull was so carefully defined and because a computer program had been developed that could deal with a wide variety of hull representations, the stage was well set for a broad exploration of many factors that were expected to influence the hull response.

The various calculations that were made on the hull of the SSB(N) 598 are given in Table II. Supporting data giving the constants for longitudinal vibration are given in Figures 6, 7 and 8. For the bending vibration calculations, the arrangement of the stations are given in Figures 9-14 in conjunction with Table III. The location of the stations is given in Table IV. The specific values of mass and stiffnesses in bending vibration are given in the computer input sheets in Appendix J.

In both the longitudinal and the bending vibrations the first computation was a repeat of the earlier one made in Reference 3 (Cases 1 and 12). In addition, a repeat of the calculations with a 10% hysteresis damping was made in order to compare with later calculations (Cases 1a and 12a). The second calculation was of a hull represented by the same number and location of stations as in Reference 3, but with each station assigned the mass and stiffness determined by the more detailed analysis of the hull. (Cases 2 and 13). The comparison between these calculations gives an indication of the possible increase in accuracy that could be gained by the laborious definition of the hull. In longitudinal vibration an exploration was made of the number of masses used to represent the ship (Cases 2, 3 and 4); of the effects of hull damping (Cases 4, 5, 6 and 7) and of the effects of changes in the difficult-to-compute stiffness between the propulsion system and the hull (Cases 4, 8 and 9). The final two runs (Cases 10 and 11) were made for use in computing the hull vibration and indicate the difference in hull response for forces on the propeller and on the hull.

For the bending vibrations there were additional factors to be investigated and so the range of calculations was larger. The difference in hull response as the propeller and

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS

Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
1	Longitudinal	DTMB Report No. 1464 Table 2	Unit Axial F. Prop. Sys. Sta. 0	Inc. in table	No damping	Check Case
2	Longitudinal	"	Unit Axial F. Hull Sta. 0	"	"	Check Case
3	Longitudinal	24 uniformly spaced masses Figure 6	Unit Axial F. Propeller	Included in Figure 6	Propeller, sprung masses, $\frac{\Delta x}{AE}$ divided by (1+0.1j)	
4	Longitudinal	16 concentr. masses Figure 7	Unit Axial F. Propeller	Included in Figure 7	Propeller, sprung masses, $\frac{\Delta x}{AE}$ divided by (1+0.1j)	
5	Longitudinal	35 concentr. masses Figure 8	"	Included in Figure 8	Propeller, and sprung masses only	
6	Longitudinal	"	"	"	Propeller, sprung masses, $\frac{\Delta x}{AE}$ divided by (1+0.04j)	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia <sup>*</sup>	Damping (coef. of imaginary term is hysteresis damping)	Remarks
7	Longitudinal	35 concentr. masses Figure 8	Unit Axial Force at Propeller	Included in Figure 8	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1 + 0.1i)$	
8	Longitudinal	"	"	"	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1 + 0.25i)$	
9	Longitudinal	35 concentr. masses, Fig. 8 increase shaft-hull stiffness from $3.3 \times 10^6$ to $6.6 \times 10^6$	"	"	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1 + 0.10i)$	
10	Longitudinal	35 concentr. masses, Fig. 8 decrease shaft-hull stiffness from $3.3 \times 10^6$ to $1.0 \times 10^6$	Unit Axial force at propeller	Included in Fig 8	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1 + 0.10i)$	
11	Longitudinal	35 concentr. masses, Fig. 8	Unit Axial force at Sta. 36	"	"	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
12	Vertical Bending	DTMB Report No. 1464 Table 1-Subm. Fig. 9 Appendix J1-3	Unit vertical force at Station 0	Included in mass	No damping	Check Case
12A	Vertical Bending	"	"	"	$\frac{1}{ET}$ and $\frac{1}{KAG}$ divided by $(1 + 0.10j)$	12 with 10% hysteresis
13	Vertical Bending	24 uniformly spaced masses, hull only Fig. 10 Appendix J4-11	Unit vertical force at Stern	$J = 0.87$ for $0 < n < 3.5$ cps $J = 0.82$ for $3.5$ cps $< n < 25.7$ cps	Sprung masses, $\frac{1}{ET}$ and $\frac{1}{KAG}$ divided by $(1 + 0.10j)$	
14	Vertical Bending	24 uniformly spaced masses + propulsion sys. Fig. 11 Appendix J12-16	"	"	Propeller, sprung masses, $\frac{1}{ET}$ and $\frac{1}{KAG}$ divided by $(1 + 0.10j)$	
15	Vertical Bending	" Figure 11 Appendix J17-20	Unit vertical force at Propeller	"	"	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
16	Vertical Bending	16 conc. masses + propulsion sys. Figure 12 Appendix J21-27	Unit vertical force at Propeller	$J = 0.87$ for $0 < n < 3.5$ cps $J = 0.82$ for $3.5$ cps $< n < 25.7$ cps	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{KAG}{EI}$ divided by $(1 + 0.10i)$	
17	Vertical Bending	35 conc. masses + propulsion sys. Figure 13 Table III - Appendix J28-36	Unit vertical force at Propeller	$J = 0.87$ for $0 < n < 3.5$ cps $J = 0.82$ for $3.5$ cps $< n < 25.7$ cps	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{KAG}{EI}$ divided by $(1 + 0.10i)$	
18	Vertical Bending	" J37-38	"	J determined for sinusoidal mode shape	"	
19	Vertical Bending	" Fig. 13 Table III Appendix J39-41	"	"	Propeller and sprung masses only	
20	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J42-44	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{KAG}{EI}$ divided by $(1 + 0.04i)$	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATION (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia <sup>*</sup>	Damping (coef. of imaginary term is hysteresis damping)	Remarks
21	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J45-47	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1 + 0.25j)$	
22	Vertical Bending	35 conc. masses + propulsion sys. hull stiffness conn. at B increased from $0.538 \times 10^6$ to $1.2 \times 10^6$ lb/in at C from $0.232 \times 10^6$ to $0.7 \times 10^6$ lb/in Fig. 13 Table III Appendix J48-50	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1 + 0.10j)$	
23	Vertical Bending	35 conc. masses + propulsion sys. hull stiffness of conn. at B decreased from $0.538 \times 10^6$ to $0.20 \times 10^6$ lb/in and at C from $0.232 \times 10^6$ to $0.10 \times 10^6$ lb/in Fig. 13 Table III Appendix J51-52	"	"	"	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
24	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J53-55	Unit moment about horizontal axis at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10i)$	
25	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J56-57	Unit vertical force at Station 54	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10i)$	
26	Vertical Bending	" Fig. 13 Table III Appendix J58-59	Unit vertical force at Station 51	"	"	
27	Vertical Bending	35 conc. masses + propulsion sys. hold all sprung masses rigidly Fig. 14 Appendix J60-63	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.1i)$	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia <sup>*</sup>	Damping (coef. of imaginary term is hysteresis damping)	Remarks
28	Longitudinal	Distributed + conc. weights + propulsion sys.	Unit axial force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.1i)$	This was not run
29	Vertical Bending	"	Unit vertical force at Propeller	"	"	This was not run
30	Vertical Bending	35 conc. masses including rotary inertia + propulsion sys. + sprung weights Fig. 13 Table III Appendix J 64-70	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.1i)$	

\* The sectional water inertia values are reduced to account for bending vibration modes by the value J. See Appendix C.

STIFFNESS $k = \frac{\text{LBS.}}{\text{IN.}}$	WEIGHT LBS.	STATION NO.	
$10^4 (1+bi) 94.34$	201,580	1	
$10^4 (1+bi) 210.08$	520,900	2	
$10^4 (1+bi) 252.53$	661,350	3	
$10^4 (1+bi) 313.48$	922,550	4	
$10^4 (1+bi) 295.86$	654,575	5	
$10^4 (1+bi) 309.11$	449,000	6	
$10^4 (1+bi) 456.62$	765,925	7	
$10^4 (1+bi) 420.17$	765,675	8	
12400	973,310	9	
5720	6900	10	
$10^4 (1+bi) 458.72$	3145	11	$C_{9,10} = 24.6$
$10^4 (1+bi) 480.77$	893,700	12	$C_{9,11} = 11.5$
$10^4 (1+bi) 480.77$	992,600	13	
$10^4 (1+bi) 480.77$	945,000	14	
$10^4 (1+bi) 480.77$	978,000	15	
$10^4 (1+bi) 456.62$	921,800	16	
11600	1,072,725	17	
11200	15,200	18	
8100	15,200	19	$C_{17,18} = 27.8$
$10^4 (1+bi) 401.61$	12,261	20	$C_{17,19} = 19.3$
6160	1,157,040	21	$C_{17,20} = 15.2$
$10^4 (1+bi) 288.18$	3,450	22	
307000	727,900	23	$C_{21,22} = 25.25$
$10^4 (1+bi) 271.74$	51,893	24	$C_{23,24} = 192$
9000	631,500	25	
20640	5,300	26	$C_{25,26} = 15.9$
$10^4 (1+bi) 251.26$	32,920	27	$C_{25,27} = 107$
$10^4 (1+bi) 281.69$	559,900	28	
7150	391,450	29	
36120	3,991	30	$C_{29,30} = 14.2$
7150	54,940	31	$C_{29,31} = 81.5$
$10^4 (1+bi) 209.21$	3,991	32	$C_{29,32} = 14.2$
$10^4 (1+bi) 176.367$	354,675	33	
$10^4 (1+bi) 158.696$	280,850	34	
$10^4 (1+bi) 94.16$	215,050	35	
6400	142,275	36	
$10^4 (1+bi) 52.91$	4,300	37	$C_{36,37} = 13.6$
$10^4 (1+bi) 3.3$	124,230	38	
$10^4 (1+bi) 10$	51,680	39	$C_{39} = 695$
$10^4 (1+bi) 10.6$	31,900	40	
$10^4 (1+bi) .10$	13,660	41	
$10^4 (1+bi) .26$	15,100	42	
$10^4 (1+bi) 12$	4,000	43	
$10^4 (1+bi) 2.77$	4,500	44	
	7,340	45	

b Represents the  
Hysteresis Damping  
C is the Concentrated  
Damping Constant  $\frac{\text{LBS.}}{\text{IN.}}$

FIG. 7 LONGITUDINAL VIBRATION  
HULL REPRESENTATION BY 25 UNIFORMLY SPACED  
CONCENTRATED MASSES

WEIGHT LBS.	STIFFNESS $K = \frac{10^6 \text{ lb}}{\text{in.}}$	STA. No
8145	$(1+bi) 1.27 \times 10^6$	1
584,813	$(1+bi) 70.82 \times 10^6$	2
1,580,247	$(1+bi) 105.87 \times 10^6$	3
1,028,411	$(1+bi) 174.22 \times 10^6$	4
1,327,915	$(1+bi) 224.72 \times 10^6$	5
1,642,546	12400	6
6,900	5720	7
3145	$(1+bi) 249.38 \times 10^6$	8
2,357,950	$(1+bi) 240.38 \times 10^6$	9
916,000	$(1+bi) 283.29 \times 10^6$	10
3,312,928	11600	11
15,200	11200	12
15,200	8100	13
12,261	6160	14
3,450	$(1+bi) 204.08 \times 10^6$	15
790,880	307,000	16
51,893	$(1+bi) 404.86 \times 10^6$	17
286,905	$(1+bi) 162.60 \times 10^6$	18
1,425,200	9000	19
5,300	20640	20
32,920	7150	21
3,991	36120	22
54,940	7150	23
3,991	$(1+bi) 56.883 \times 10^6$	24
669,413	$(1+bi) 91.575 \times 10^6$	25
201,169	6400	26
4,300	$(1+bi) 88.889 \times 10^6$	27
89,941	$(1+bi) 41.841 \times 10^6$	28
33,730	$(1+bi) 3.3 \times 10^6$	29
51,680	$(1+bi) 10 \times 10^6$	30
31,900	$(1+bi) 10.6 \times 10^6$	31
13,660	$(1+bi) .1 \times 10^6$	32
15,100	$(1+bi) 26 \times 10^6$	33
4,000	$(1+bi) 12 \times 10^6$	34
4,500	$(1+bi) 2.77 \times 10^6$	35
7,340		36

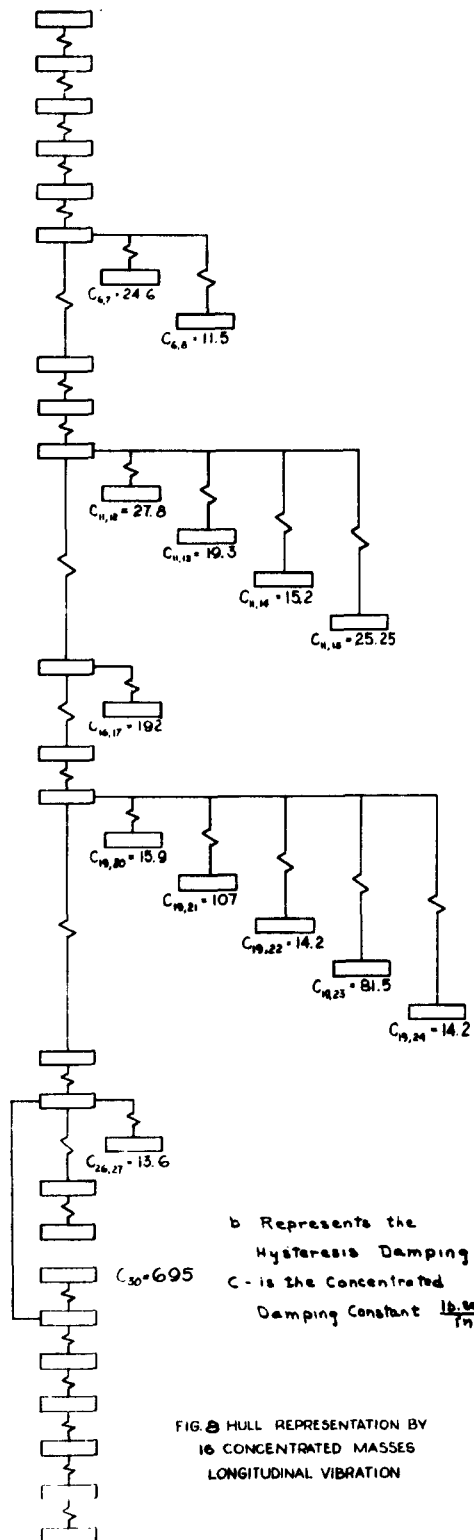
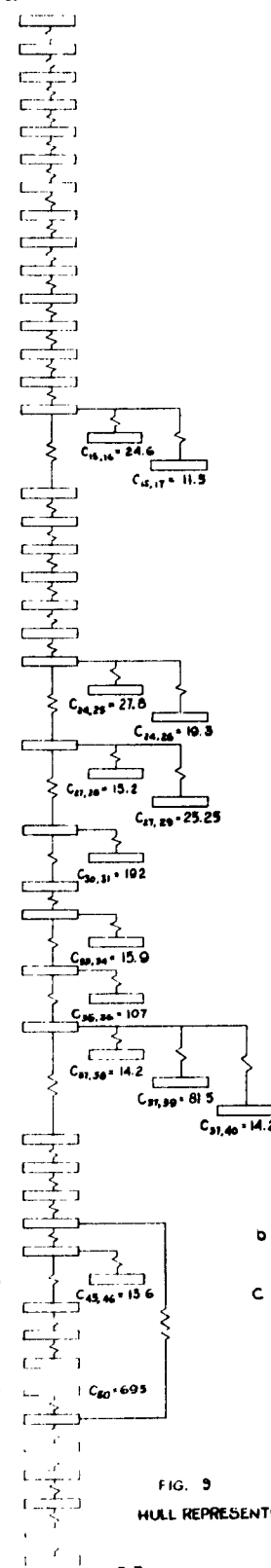


FIG. 2 HULL REPRESENTATION BY 16 CONCENTRATED MASSES LONGITUDINAL VIBRATION

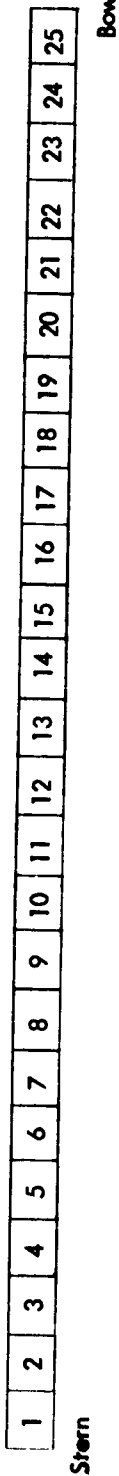
# STIFFNESS WEIGHT STA. NO.

$k = 1.89 \times 10^6$	8,145	1
$98.9 \times 10^6$	70,394	2
$166.6 \times 10^6$	158,570	3
$161.53 \times 10^6$	355,899	4
$253.81 \times 10^6$	293,623	5
$250.6 \times 10^6$	861,205	6
$408.16 \times 10^6$	425,419	7
$719.42 \times 10^6$	341,547	8
$649.35 \times 10^6$	387,288	9
$396.83 \times 10^6$	299,576	10
$505.05 \times 10^6$	317,196	11
$806.45 \times 10^6$	492,448	12
$757.58 \times 10^6$	518,271	13
$653.89 \times 10^6$	583,650	14
$625.0 \times 10^6$	534,144	15
12400	6,900	16
5720	3,145	17
$729.93 \times 10^6$	524,752	18
$515.46 \times 10^6$	599,000	19
$450.43 \times 10^6$	813,450	20
$598.80 \times 10^6$	945,000	21
$337.63 \times 10^6$	916,000	22
$363.64 \times 10^6$	786,000	23
$465.12 \times 10^6$	1,010,291	24
11800	15,200	25
11200	15,200	26
$404.85 \times 10^6$	1,516,637	27
8100	12,261	28
6160	3,450	29
$465.12 \times 10^6$	790,880	30
307000	51,893	31
$404.85 \times 10^6$	286,905	32
$523.56 \times 10^6$	484,457	33
9000	5,300	34
$233.85 \times 10^6$	562,527	35
20640	32,920	36
$276.24 \times 10^6$	378,235	37
7180	3,991	38
36120	54,940	39
7150	3,991	40
$132.67 \times 10^6$	325,238	41
$134.95 \times 10^6$	226,996	42
$233.85 \times 10^6$	117,179	43
$216.45 \times 10^6$	101,375	44
$485.44 \times 10^6$	55,308	45
6400	4,300	46
$174.83 \times 10^6$	44,486	47
$180.83 \times 10^6$	89,941	48
$418.41 \times 10^6$	33,730	49
$3.3 \times 10^6$	51,680	50
$10 \times 10^6$	31,900	51
$10.6 \times 10^6$	13,660	52
$.10 \times 10^6$	15,100	53
$26 \times 10^6$	4,000	54
$12 \times 10^6$	4,500	55
$2.77 \times 10^6$	7,340	56



d Represents the  
Hysteresis Damping  
C is the Concentrated  
Damping Constant  $\frac{1b, 200}{in}$

FIG. 9 LONGITUDINAL VIBRATION  
HULL REPRESENTATION BY 36 CONCENTRATED MASSES

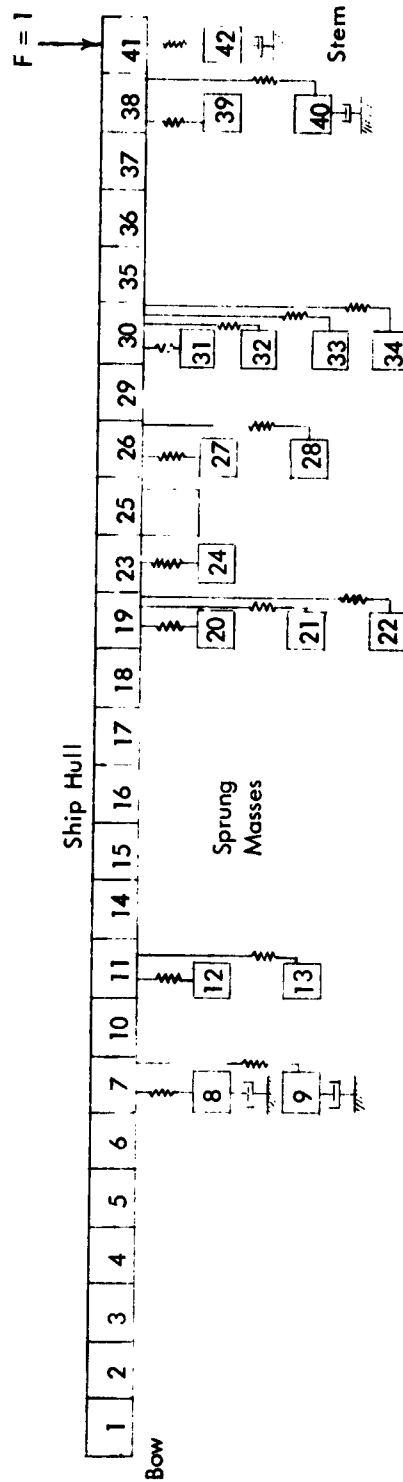


SSB(N) 598

Vertical Bending Vibration Calculations

Case 12 Ship Representation	25 Uniformly Spaced Masses Including Water Inertia Mass and Stiffness as Given in DTMB Report 1464 Excitation at the Stern (Sta. 1) No Damping
Case 12A	Identical but with 10% Hysteresis Damping

Figure 10 - Ship Representation for Calculation



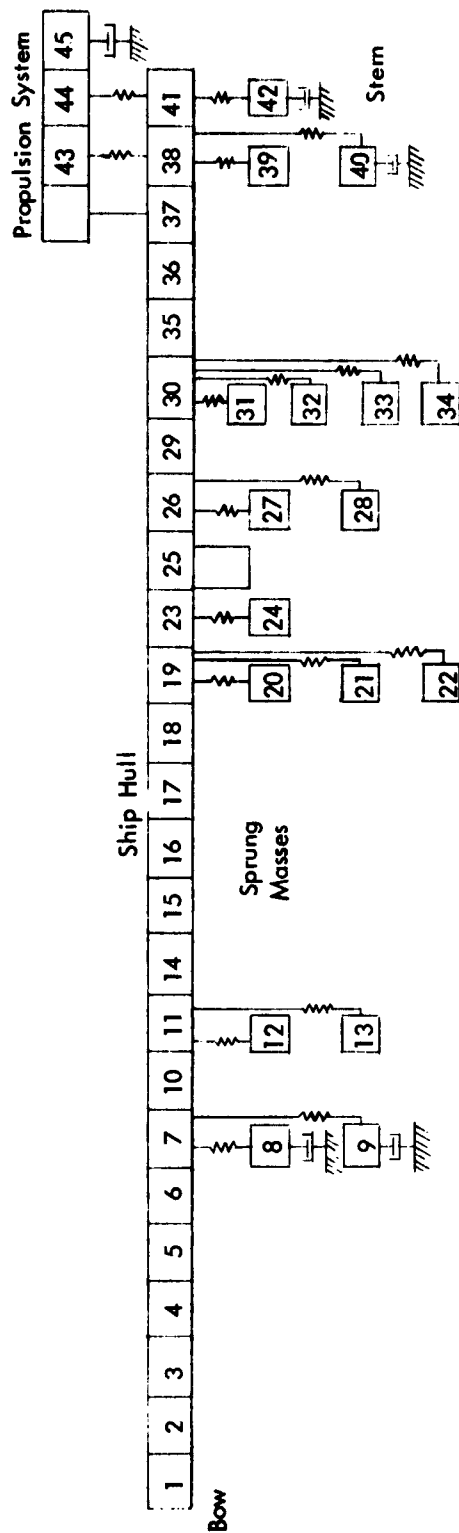
SSB(N) 598

### Vertical Bending Vibration Calculations

Case 13 Ship Representation	24 Uniformly Spaced Masses
	Damped Sprung Masses
	Water Inertia $J = 0.87$ for $0 < n < 3.5$ cps $= 0.82$ for $3.5 < n$ cps

Excitation at Stern (Sta. 41)  
10% Hysteresis Damping

Figure 11 - Ship Representation for Calculation



SSB(N) 598

### Vertical Bending Vibration Calculations

Case 14  
Ship  
Representation

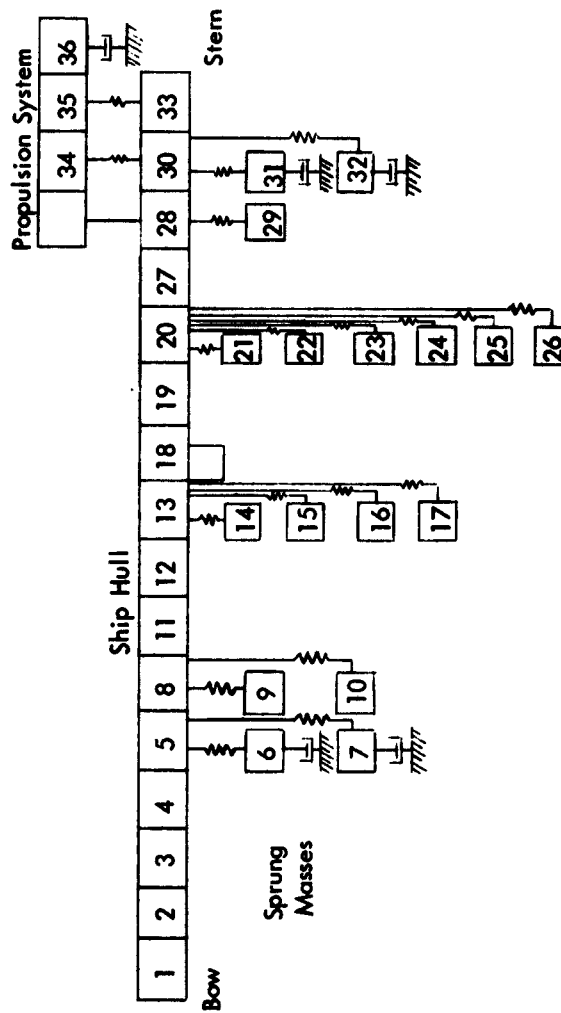
24 Uniformly Spaced Masses  
Damped Sprung Masses, Propulsion System  
Water Inertia  $J = 0.82$  for  $0 < n < 3.5$  cps;  $= 0.87$  for  $3.5 < n$  cps

Excitation at Stern (Sta. 41)  
10% Hysteresis Damping

Case 15  
Ship  
Representation

Identical with Case 14 except Excitation is at the Propeller (Sta. 45)

Figure 12 - Ship Representation for Calculation

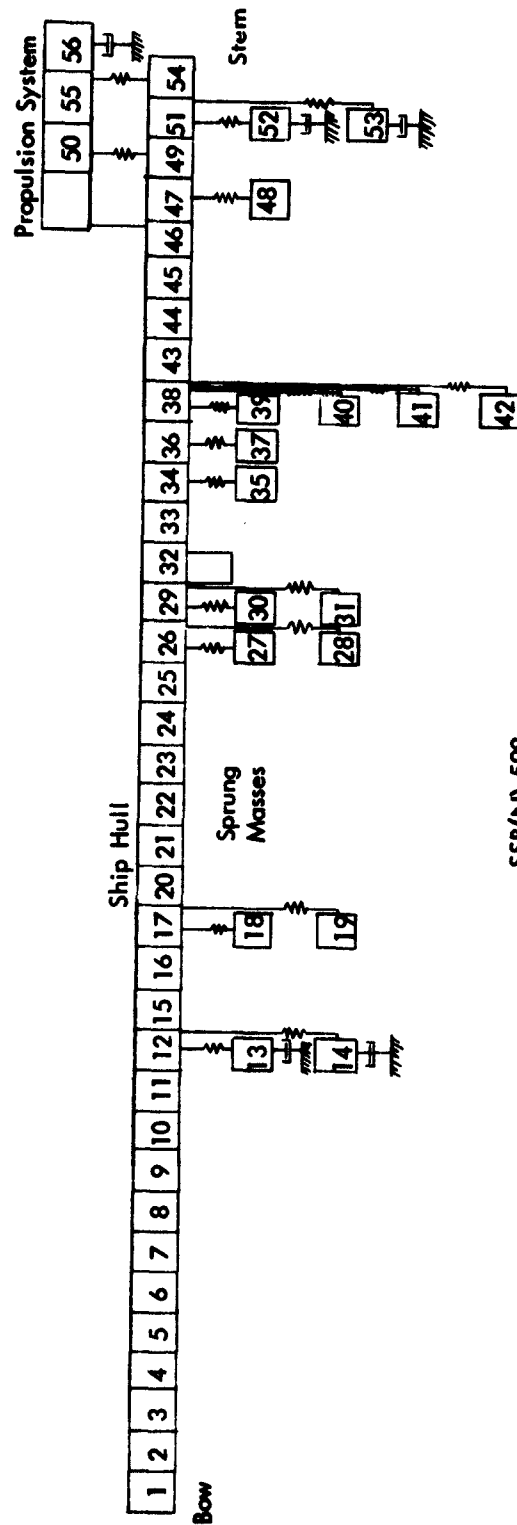


SSB(N) 598

### Vertical Bending Vibration Calculation

- |                |   |
|----------------|---|
| Case 16        | 16 Concentrated Masses  |
| Ship           | Damped Sprung Masses; Propulsion System                                     |
| Representation | Water Inertia, $J = 0.87$ for $0 < n < 3.5$ cps, $= 0.82$ for $3.5 < n$ cps |
|                | Excitation at the Propeller (Sta. 36)                                       |
|                | 10% Hysteresis Damping  |

Figure 13 - Ship Representation for Calculation



### Vertical Bending Vibration Calculations

Cases 17-26  
and 30  
Ship  
Representation

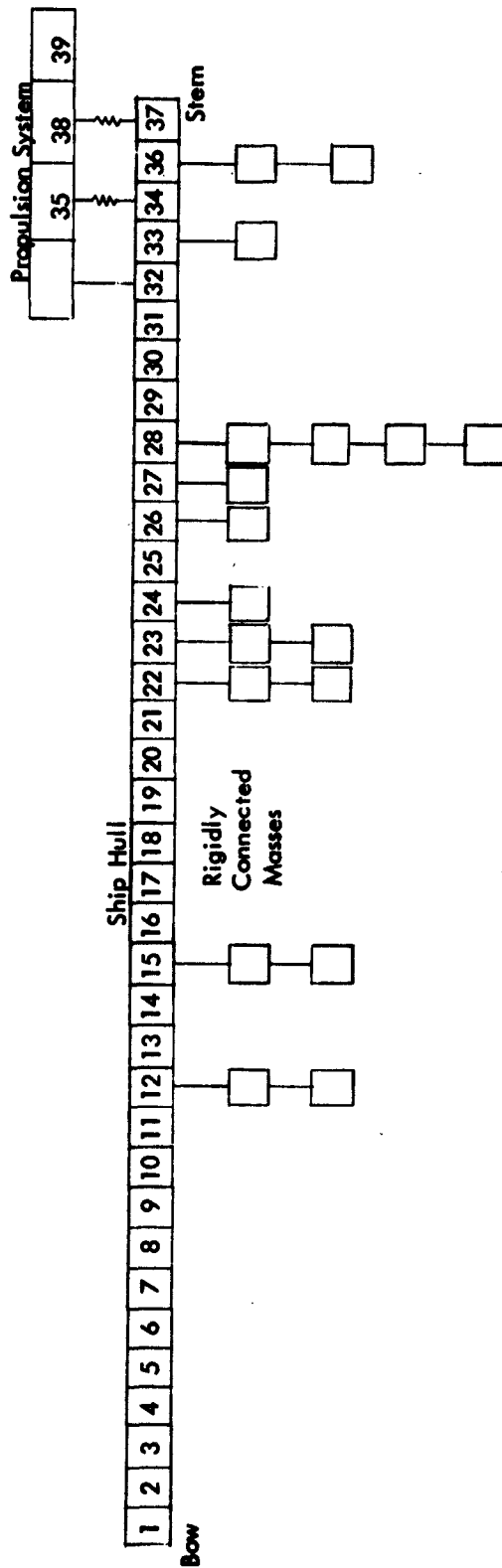
35 Concentrated Masses  
Damped Sprung Masses - Propulsion System  
Water Inertia - Representation Varies  
Damping - Representation Varies  
Excitation - Various Locations

See  
Following Table

Figure 14 - Ship Representation for Calculation

**TABLE III - 35 MASS REPRESENTATION - VARIATIONS OF VERTICAL BENDING  
VIBRATION CALCULATIONS**

Case No.	Water Inertia	Hull Damping	Excitation	Other
17	$J = 0.87$ for $0 < n < 3.5$ $= 0.82$ for $3.5 < n$	10% Hysteresis	Vertical force at Propeller (Sta. 56)	
18 Standard	J determined for sinusoidal mode shape	10% Hysteresis	Vertical force at Propeller (Sta. 56)	
19	Standard	No hull damping	Standard	
20	Standard	4% Hysteresis	Standard	
21	Standard	25% Hysteresis	Standard	
22	Standard	Standard	Standard	Stiffened connections between propulsion system and hull
23	Standard	Standard	Standard	Softened connections between propulsion system and hull
24	Standard	Standard	Moment about the horizontal axis at the propeller	
25	Standard	Standard	Vertical force at stern (Sta. 54)	
26	Standard	Standard	Vertical force forward of the stern (Sta. 51)	
30	Standard	Standard	Standard	Include rotary inertia of the hull cross-section



SSB(N) 598

### Vertical Bending Vibration Calculations

Case 27  
Ship  
Representation

35 Concentrated Masses  
Sprung Masses Assumed Rigidly Connected  
Water Inertia J Based on Sinusoidal  
Mode Shape

Excitation at Propeller (Sta. 39)  
10% Hysteresis Damping

Figure 15 - Ship Representation for Calculation

**TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM  
FORWARD PERPENDICULAR**  
(s) indicates sprung mass

Case No. Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17-26 30	27
1.	Prop.	7.5	1.5	1.5	367.5	7.5	7.5	1.5	1.5	1.5
2	Prop. shaft	22.5	12	6	352.5	22.5	22.5	12	6	6
3	Prop. shaft	37.5	45	12	337.5	37.5	37.5	45	12	12
4	Thrust brg.	52.5	69	21	322.5	52.5	52.5	69	21	21
5	Pwr. plant	67.5	98	30	307.5	67.5	67.5	98	30	30
6	367.5	82.5	127	45	292.5	82.5	82.5	98	45	45
7	352.5	97.5	127(s)	55	277.5	97.5	97.5	98(s)	55	55
8	337.5	112.5	127(s)	62	262.5	97.5(s)	97.5(s)	127	62	62
9	322.5	127.5	155	69	247.5	97.5(s)	97.5(s)	127(s)	69	69
10	307.5	127.5(s)	185	80	232.5	112.5	112.5	127(s)	80	80
11	292.5	127.5(s)	210	90	217.5	127.5	127.5	155	90	90
12	277.5	142.5	210(s)	98	202.5	127.5(s)	127.5(s)	185	98	98
13	262.5	157.5	210(s)	107	187.5	127.5(s)	127.5(s)	210	98(s)	107
14	247.5	172.5	210(s)	117	172.5	142.5	142.5	210(s)	98(s)	117
15	232.5	187.5	210(s)	127	157.5	157.5	157.5	210(s)	107	127
16	217.5	202.5	250	127(s)	142.5	172.5	172.5	210(s)	117	136
17	202.5	217.5	250(s)	127(s)	127.5	187.5	187.5	210(s)	127	145

TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM (continued)  
FORWARD PERPENDICULAR  
 (s) indicates sprung mass

Case No. Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17-26 30	27
18	187.5	217.5(s)	259	136	112.5	202.5	202.5	250	127(s)	155
19	172.5	217.5(s)	280	145	97.5	217.5	217.5	259	127(s)	169
20	157.5	217.5(s)	280(s)	155	82.5	217.5(s)	217.5(s)	280	136	185
21	142.5	232.5	280(s)	169	67.5	217.5(s)	217.5(s)	280(s)	145	197
22	127.5	232.5(s)	280(s)	185	52.5	217.5(s)	217.5(s)	280(s)	155	210
23	112.5	247.5	280(s)	197	37.5	232.5	232.5	280(s)	169	228
24	97.5	247.5(s)	280(s)	210	22.5	232.5(s)	232.5(s)	280(s)	185	250
25	82.5	262.5	326	210(s)	7.5	247.5	247.5	280(s)	197	259
26	67.5	262.5(s)	348	210(s)		262.5	262.5	280(s)	210	264
27	52.5	262.5(s)	348(s)	228		262.5(s)	262.5(s)	326	210(s)	280
28	37.5	277.5	360	228(s)		262.5(s)	262.5(s)	348	210(s)	295
29	22.5	292.5	372	228(s)		277.5	277.5	348(s)	228	315
30	7.5	292.5(s)	Prop.	250		292.5	292.5	360	228(s)	326
31		292.5(s)	Thrust brg.	250(s)		292.5(s)	292.5(s)	360(s)	228(s)	335
32		292.5(s)	L. S. Gr.	259		292.5(s)	292.5(s)	360(s)	250	343
33		307.5	L. S. pinion	264		292.5(s)	292.5(s)	372	259	348
34		322.5	H. S. Gr.	264(s)		292.5(s)	292.5(s)	Shaft Seal	264	355.42

TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM (continued)  
 FORWARD PERPENDICULAR  
 (s) indicates sprung mass

Case No. Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17-26 30	27
35		337.5	H. S. pinion	280		307.5	307.5	Stern Brg.	264(s)	Shaft Seal
36		352.5	Turb.	280(s)		322.5	322.5	Prop.	280	360
37		352.5(s)		295		337.5	337.5		280(s)	372
38		367.5		295(s)		352.5	352.5		295	Stem Brg.
39		Prop.		295(s)		352.5(s)	352.5(s)		295(s)	Prop.
40		Thrust Brg.		295(s)		352.5(s)	352.5(s)		295(s)	
41		Low speed Gr. Hub.		315		367.5	367.5		295(s)	
42		Low speed Pinions		326		367.5(s)	367.5(s)		295(s)	
43		H. S. gear		335			Shaft Seal		315	
44		H. S. Pinions		343			Stern Brg.		326	
45		Turb.		348			Prop.		335	
46				348(s)					343	
47				355.42					348	

**TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM (continued)**  
**FORWARD PERPENDICULAR**  
 (s) indicates sprung mass

Case No. Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17-26 30	27
48				360					348(s)	
49				372					355.42	
50				Prop.					Shaft seal	
51				Thrust Brig.					360	
52				L. S. Gr. Hub.					360(s)	
53				L. S. Pinions					360(s)	
54				H. S. Gear					372	
55				H. S. Pinions					Stem Brig.	
56				Turb.					Prop.	

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its connection to the hull is brought into consideration is shown by a comparison between Cases 13 and 14 where the excitation remains on the hull and with Case 15 where the excitation is transferred to the propeller. The influence of the number of masses used to define the ship is shown by a comparison of Cases 15, 16 and 17. A comparison between runs 17 and 18 indicates the effects upon the hull response of different assumptions for the reduction of the sectional water inertia to allow for the modal pattern of the hull vibration (See Appendix C). The effects of hull damping are determined through Cases 18, 19, 20 and 21. The effects of changes in the stiffness of the connections between the propulsion system and the hull are shown by Cases 18, 22 and 23. Case 24 is included to give a base for estimating the hull response to a moment at the propeller and Cases 25 and 26 to forces acting on the hull. Case 27 is included for comparison with Case 18 to show the influences of the sprung masses (whose gross weight is very small) upon the hull response and Case 30 by comparison with Case 18 shows the importance of including the rotary inertia of the hull cross sections in making calculations.

## VII RESULTS OF CALCULATIONS

The results of the calculations, which are in the form of deflections at each station for a range of frequencies from 1 to 25 cps, are best shown by curves of hull motion per pound of excitation force at selected positions and phase angle relative to the excitation. These curves are presented in Figures 16 through 60. From a study of these curves it is possible to determine the answers to some of the questions raised in the previous section of this report.

### 1. Longitudinal Vibration:

In comparing the results of the several longitudinal vibration calculations there are several lessons to be noted but there does not appear to be any trend that could not be expected. Case 7 (Figures 21 and 22) is the standard to which other cases are compared.

In comparing Case 1 with Case 7, the first noticeable difference is the larger

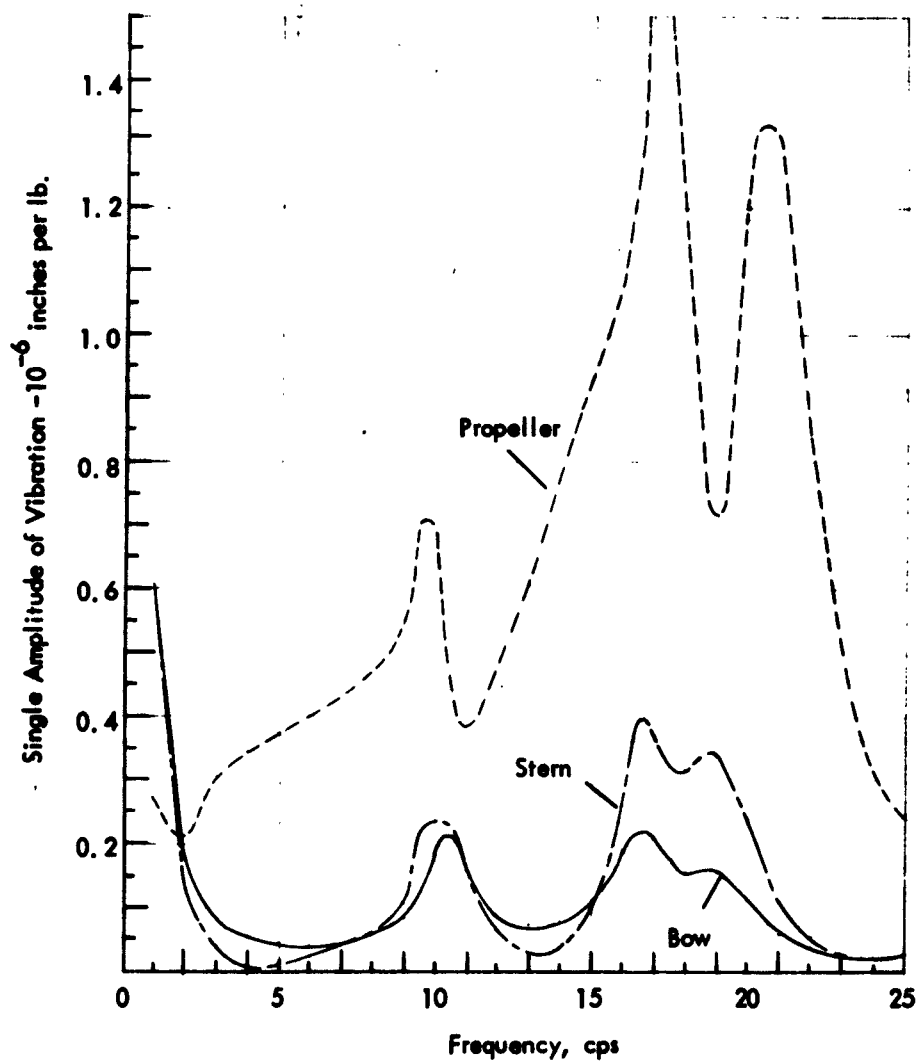


Figure 16-Response Curves in Longitudinal Vibration

Case 1a, Excitation at Propeller. 10% Hysteresis Damping  
Mass and Stiffness taken from DTMB Report 1464

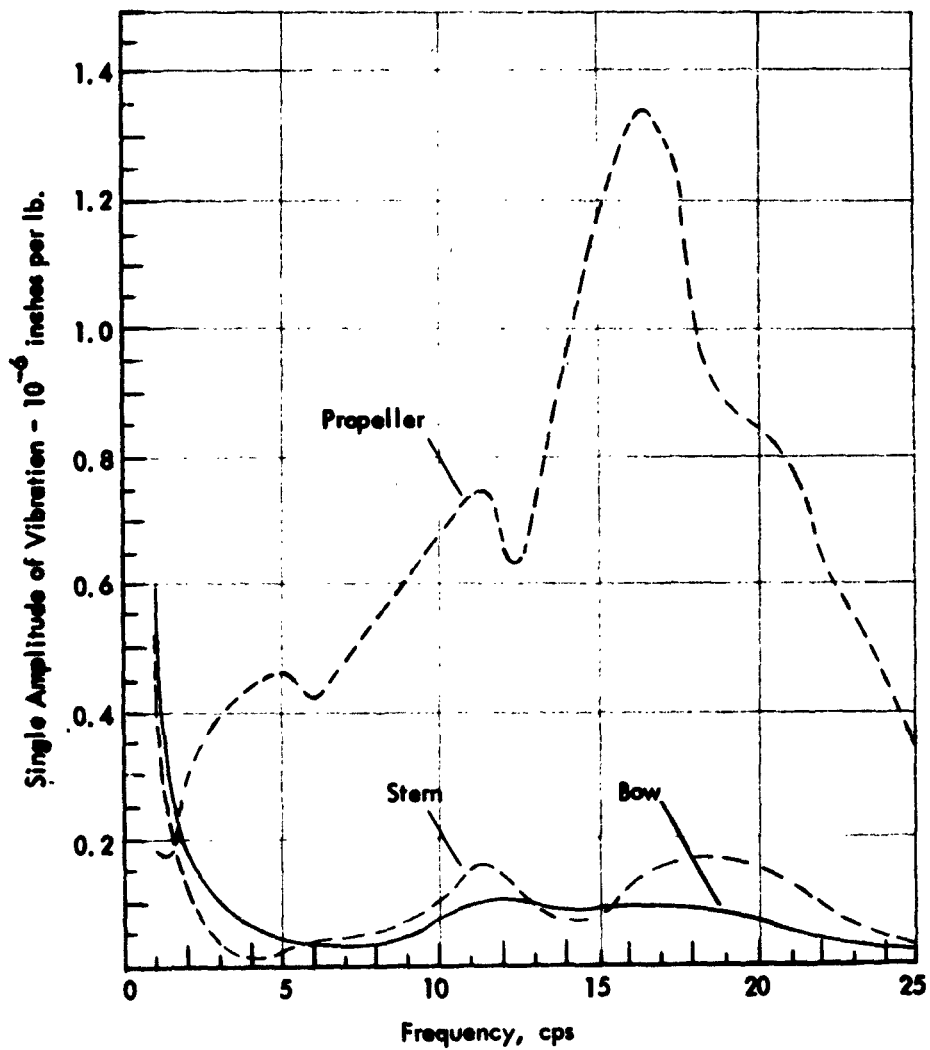


Figure 17 - Response Curves in Longitudinal Vibration

Case 3. Excitation at Propeller. 10% Hysteresis Damping  
24 Uniformly Spaced Masses, Sprung Masses, Propulsion System

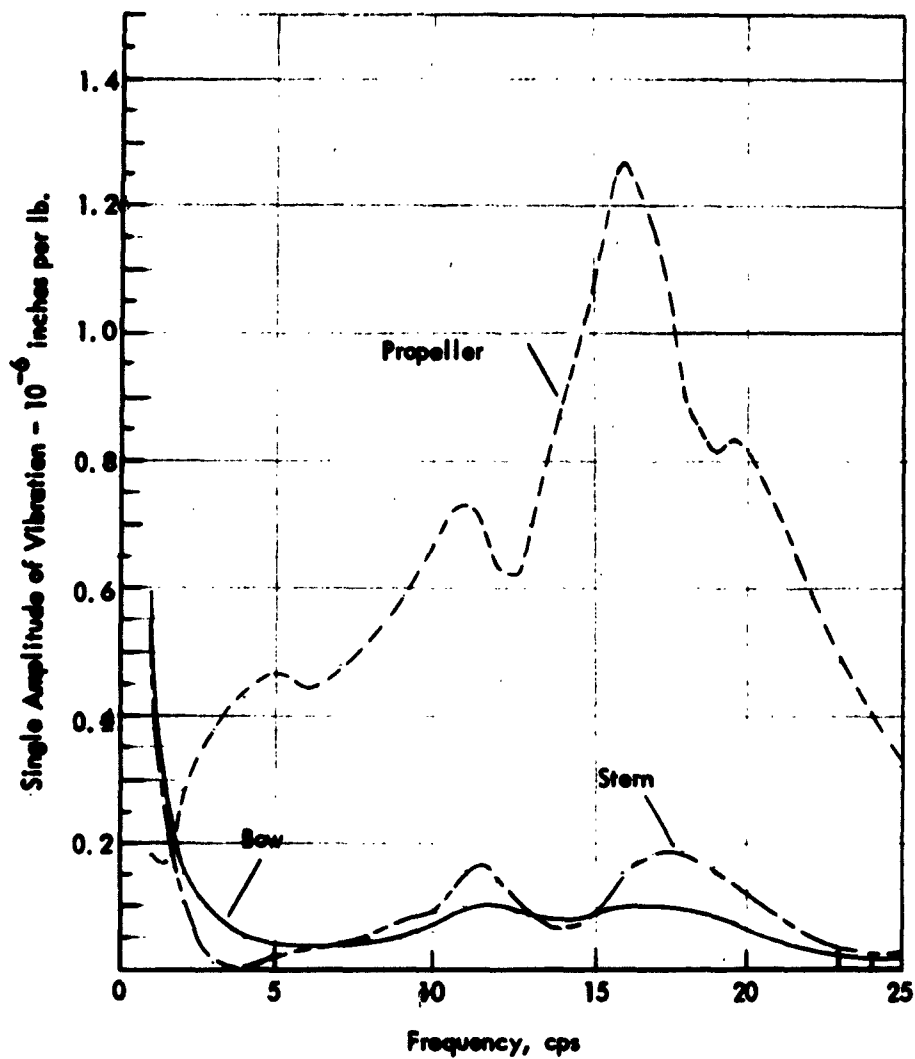


Figure 18 - Response Curves in Longitudinal Vibration

Case 4. Excitation at Propeller. 10% Hysteresis Damping  
16 Concentrated Masses, Sprung Masses, Propulsion System

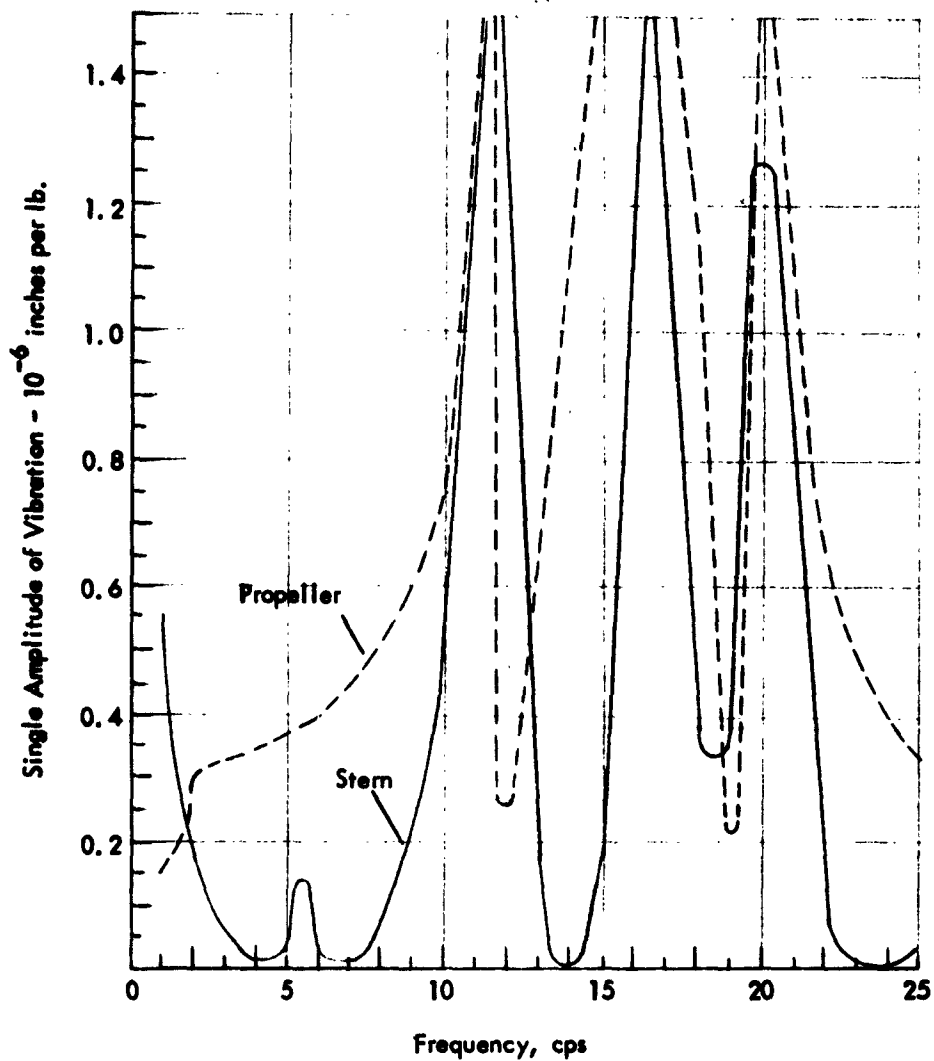


Figure 19 - Response Curves in Longitudinal Vibration

Case 5. Excitation at Propeller. No Hysteresis Damping  
35 Concentrated Masses, Sprung Masses, Propulsion System

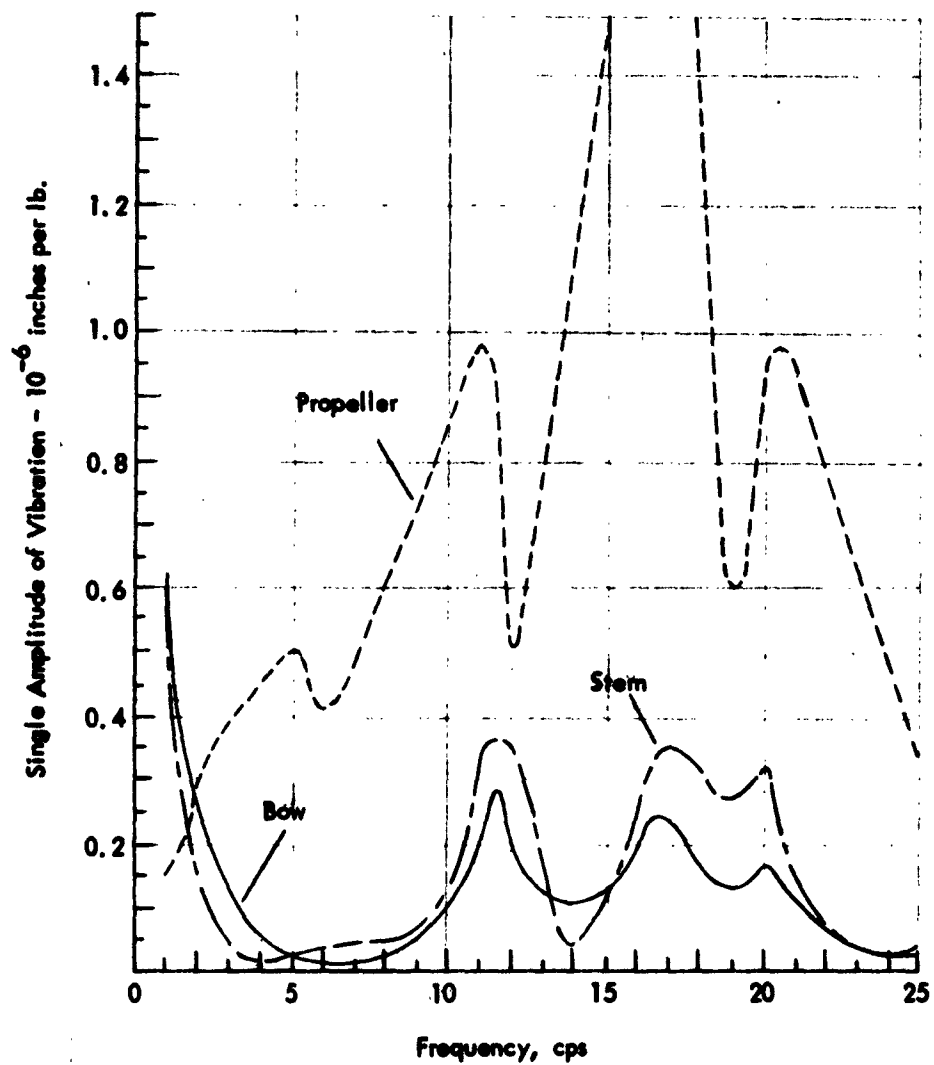


Figure 20 - Response Curves in Longitudinal Vibration

Case 6. Excitation at Propeller. 4% Hysteresis Damping  
35 Concentrated Masses, Sprung Masses, Propulsion System

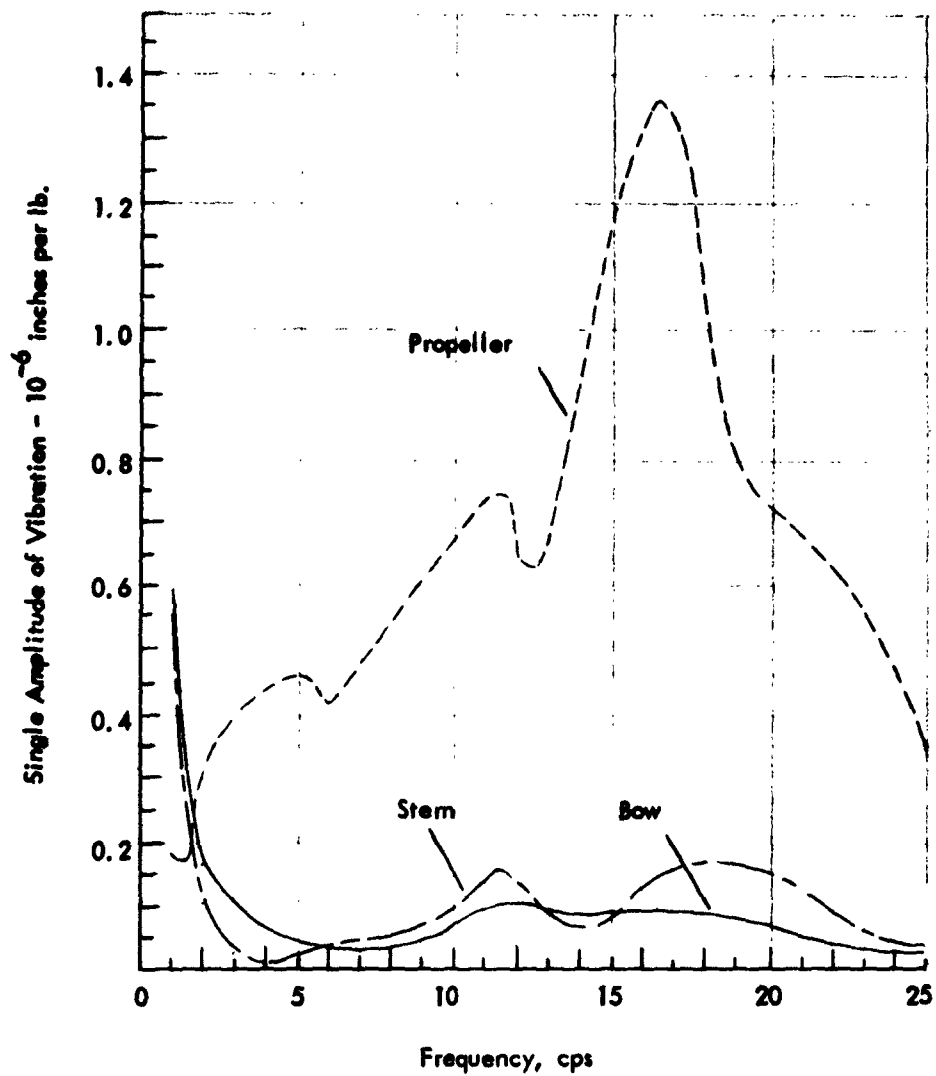


Figure 21 - Response Curves in Longitudinal Vibration

Case 7. Excitation at Propeller. 10% Hysteresis Damping  
35 Concentrated Masses, Sprung Masses, Propulsion System

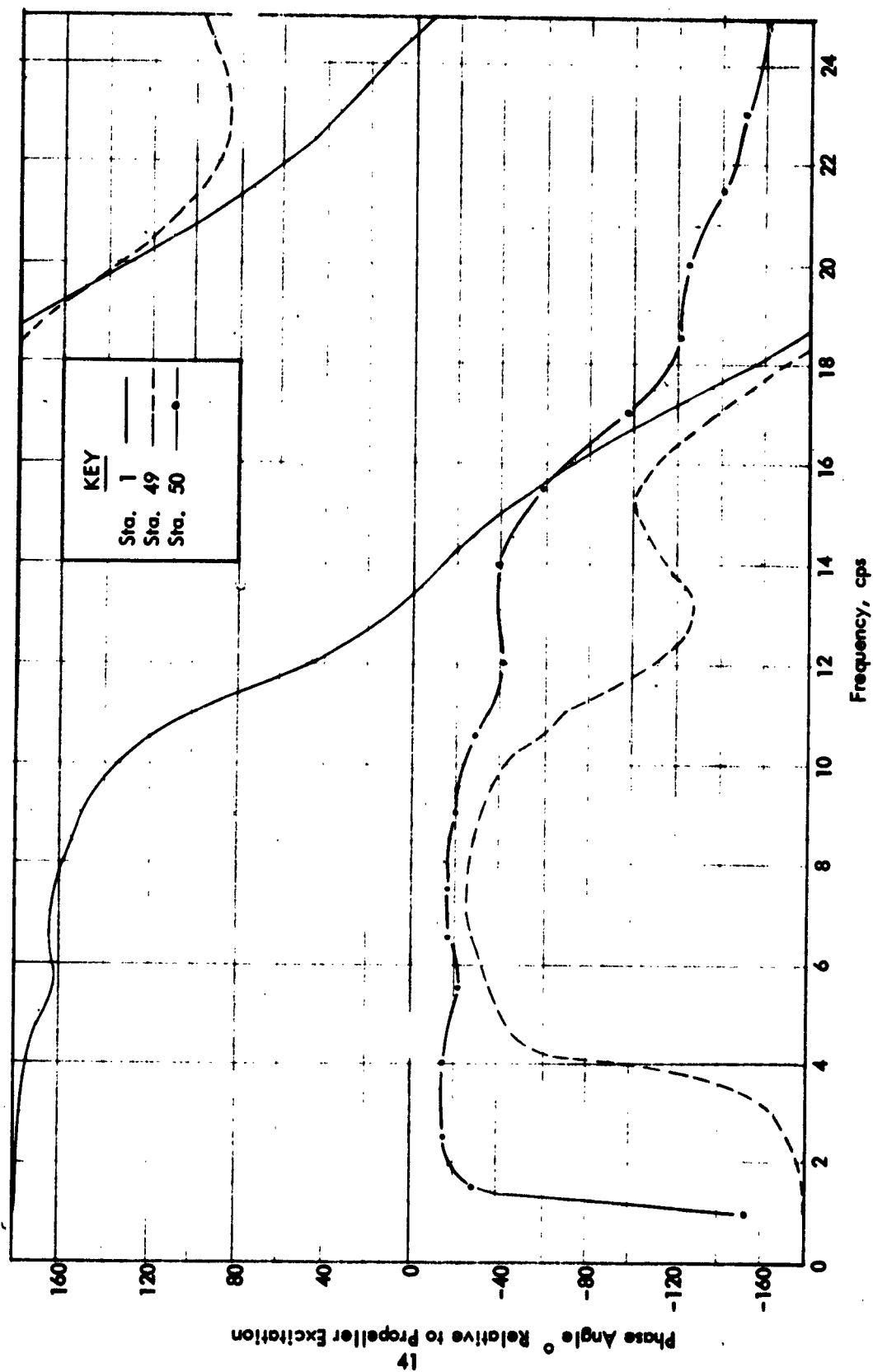


Figure 22 - Longitudinal Vibration - Phase Angle Relative to Propeller Excitation, Case 7

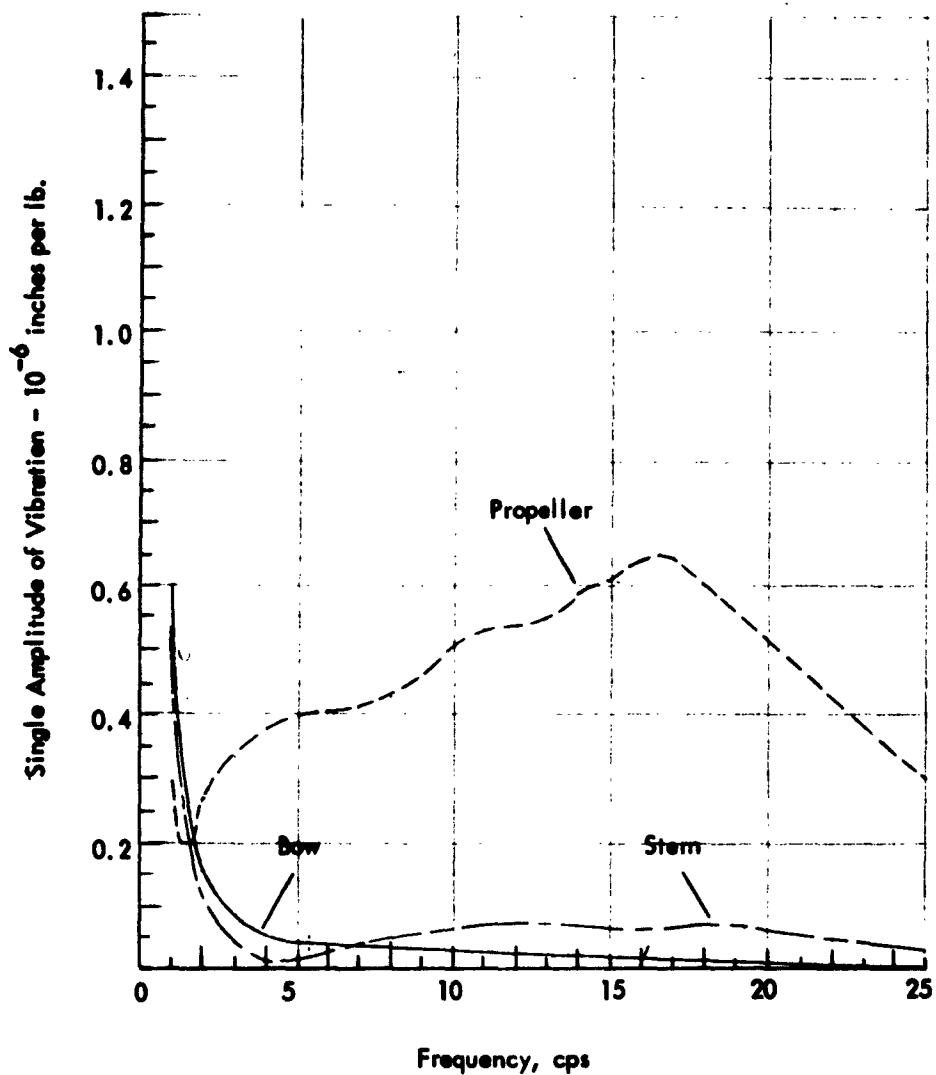


Figure 23 - Response Curves in Longitudinal Vibration

Case 8. Excitation at Propeller. 25% Hysteresis Damping  
35 Concentrated Masses, Sprung Masses, Propulsion System

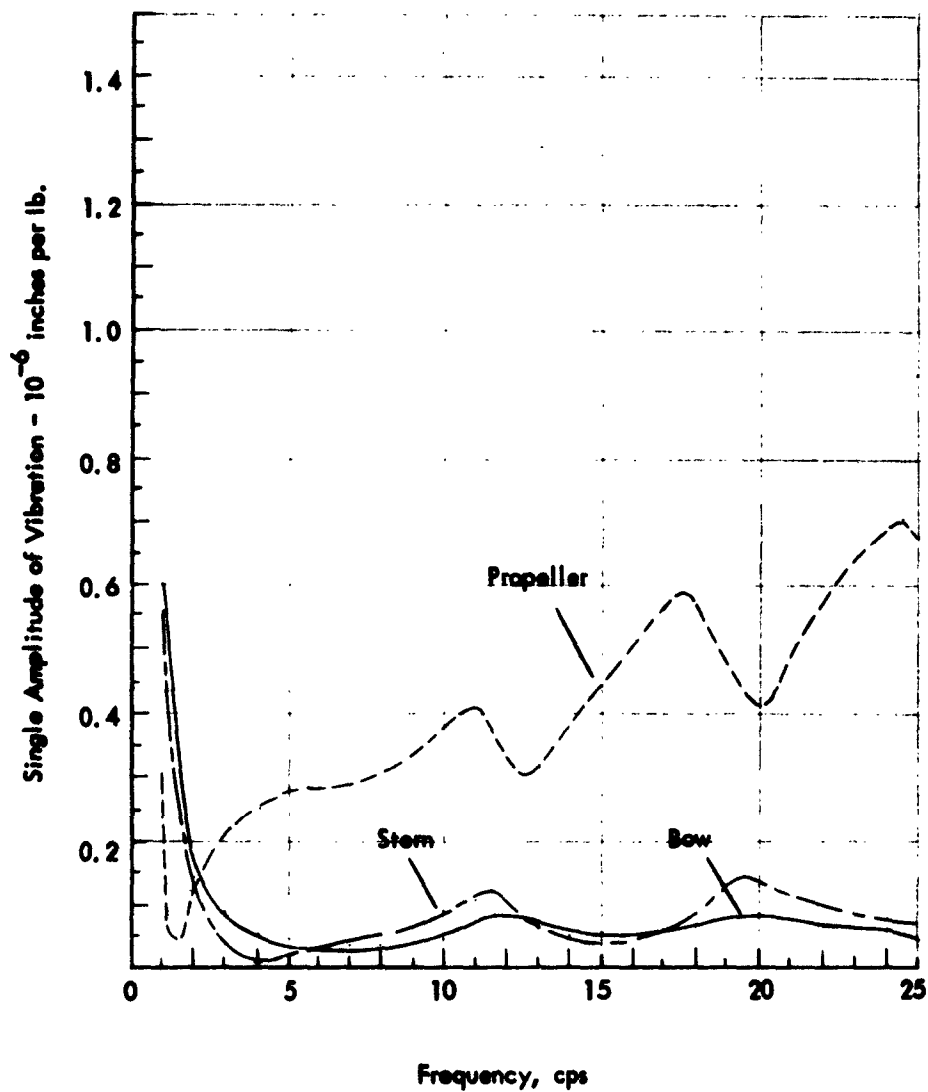


Figure 24 - Response Curves in Longitudinal Vibration

Case 9. Excitation at Propeller. 10% Hysteresis Damping  
 35 Concentrated Masses, Sprung Masses, Propulsion System, Stiffness of Connection  
 between Hull and Thrust Bearing twice that of Case 7.

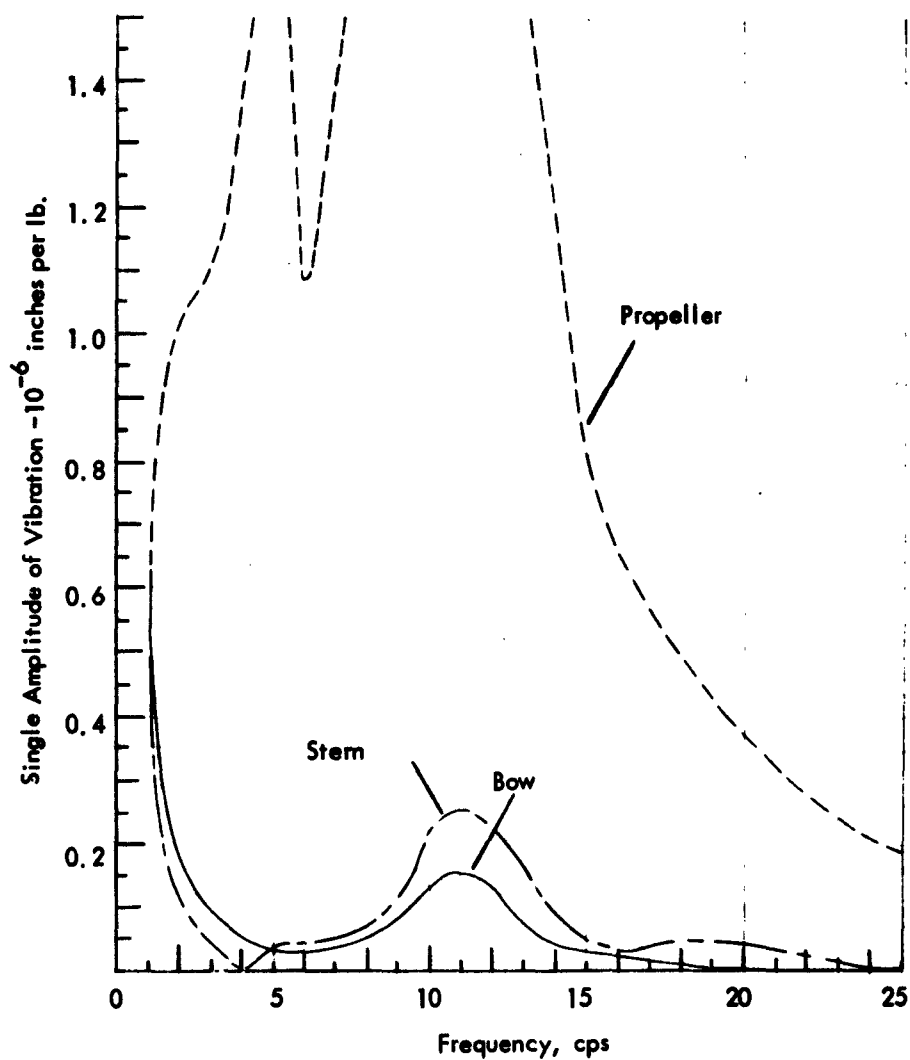
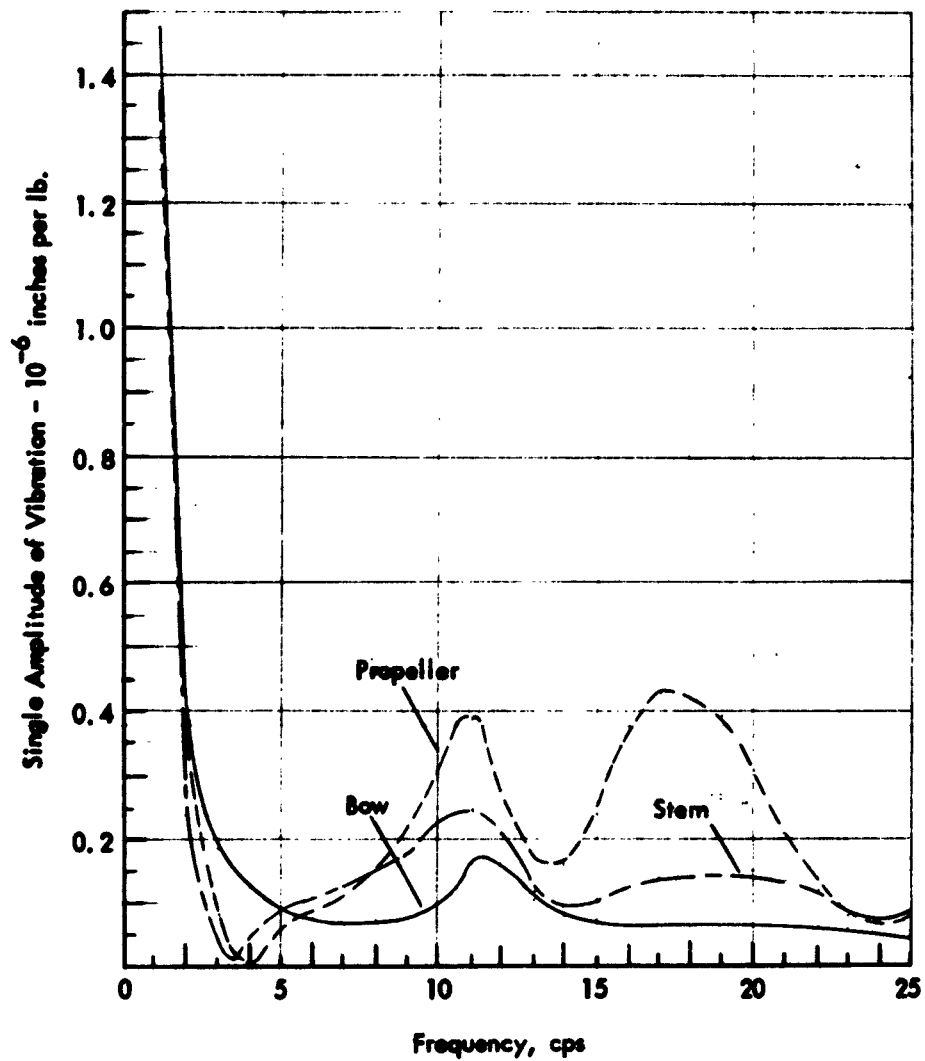


Figure 25-Response Curves in Longitudinal Vibration

Case 10. Excitation at Propeller. 10% Hysteresis Damping

35 Concentrated Masses, Sprung Masses, Propulsion System, Stiffness of Connection between Hull and Thrust Bearing, 30% of that in Case 7.



**Figure 26 - Response Curves in Longitudinal Vibration**

**Case 11. Excitation at Stern. 10% Hysteresis Damping  
35 Concentrated Masses, Sprung Masses, Propulsion System**

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amplitude of hull motion in Case 1. The resonant frequencies occur at about the same frequencies (except that the propeller in Case 7 shows a weak resonance at about 5 cycles that is not present in Case 1.) This corresponds to a mode in which the high speed parts of the system (turbines and high speed gears) vibrate against the propeller and the hull through the soft longitudinal stiffness of the bull gear web. The differences in hull amplitude can be explained by the addition of propeller damping and the damping of the sprung masses to the general hysteresis damping of the hull in Case 7.

A comparison of the effect of the number of stations used in defining the hull can be obtained by considering Cases 3 (24 masses), 4 (16 masses), and 7 (35 masses). The hull motion is the same in Cases 3 and 7 but differs some in Case 4. The propeller motion (which is really of secondary interest) is identical in Cases 3 and 7 except for a slight hump at about 20.5 cps in Case 3. There are slightly more differences between Case 4 and Cases 3 and 7 in the form of slightly sharpened resonances, but these are not large. It is probably better to use more than 16 stations in defining the hull.

When Cases 5, 6, 7, and 8 are compared, it becomes clear that the damping of the propeller and the sprung masses is not sufficient to keep the amplitude of hull vibration to reasonable limits. However, the addition of 4% of hysteresis damping is very effective. 10% and 25% damping make the submarine dead.

Cases 10, 7 and 9 illustrate the effects of the thrust bearing stiffness upon the hull response. In Cases 7 and 9 the effect of the thrust bearing stiffness in the amplitude of motion of the propeller is large but its influence on hull motion is small. However, Case 10 shows that when resonant frequency of the propeller on its thrust bearing matches a natural frequency of the hull, the amplitude of the hull motions can be increased. When this is recognized, it becomes clear that the amplitude of the hull motion caused by relatively high frequency ( $>15$  cps) propeller excitation is strengthened by propulsion systems that have high axial resonant frequencies (i.e., stiff thrust blocks). It is probable that the very low natural frequency that can be obtained by supporting the thrust shoes on an air spring, could be very effective in attenuating the amplitude of hull vibration in the

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axial modes.

Case 11 was run primarily for estimating the amplitude of hull motion under service conditions. However, when compared with Case 7, it illustrates that a harmonic force generated at the propeller will generally but not always give smaller ship motions than one generated on the surface of the hull.

### 2. Vertical Bending Vibration

The most conspicuous influence upon the bending vibration is the inclusion of the propulsion system as a sub-system in the calculations. This influence is first strongly shown even when the excitation is on the ship hull by comparing Cases 13 and 14 and when the excitation is transferred to the propeller, Case 15, results in a hull response that is entirely different from that obtained when the propulsion sub-system is not considered.

A comparison among Cases 15, 16 and 17 in which the hull is represented by 25, 16 and 35 concentrated masses shows that the predominant propeller frequency is much lower in Case 15 than in Case 16 or 17. This is because of the difficulty of tying the propulsion sub-system into the hull properly when required to work with regular station spacings.

In comparing Case 16 with Case 17, it will be noted that although the resonances occur at about the same frequencies, the response is different even at low frequencies. This result was not expected.

In comparing Cases 17 and 18 in which the treatment of the water inertia effects is changed (See Appendix C for an explanation of this) it is clear that no great change occurs because of the reduced inertia associated with the sinusoidal approximation but that the vibration does carry farther into the hull at the higher frequencies. Thus the amplitude of vibration at the bow at 16 cps is greater for Case 18 than for Case 17.

The effects of hull damping are shown in Cases 19 (no damping) 20 (4% damping) 18 (10% damping) and 21 (25% damping). Referring to Case 19 it is clear that the hull resonances are quite sharp and that the sprung masses and the propeller damping are

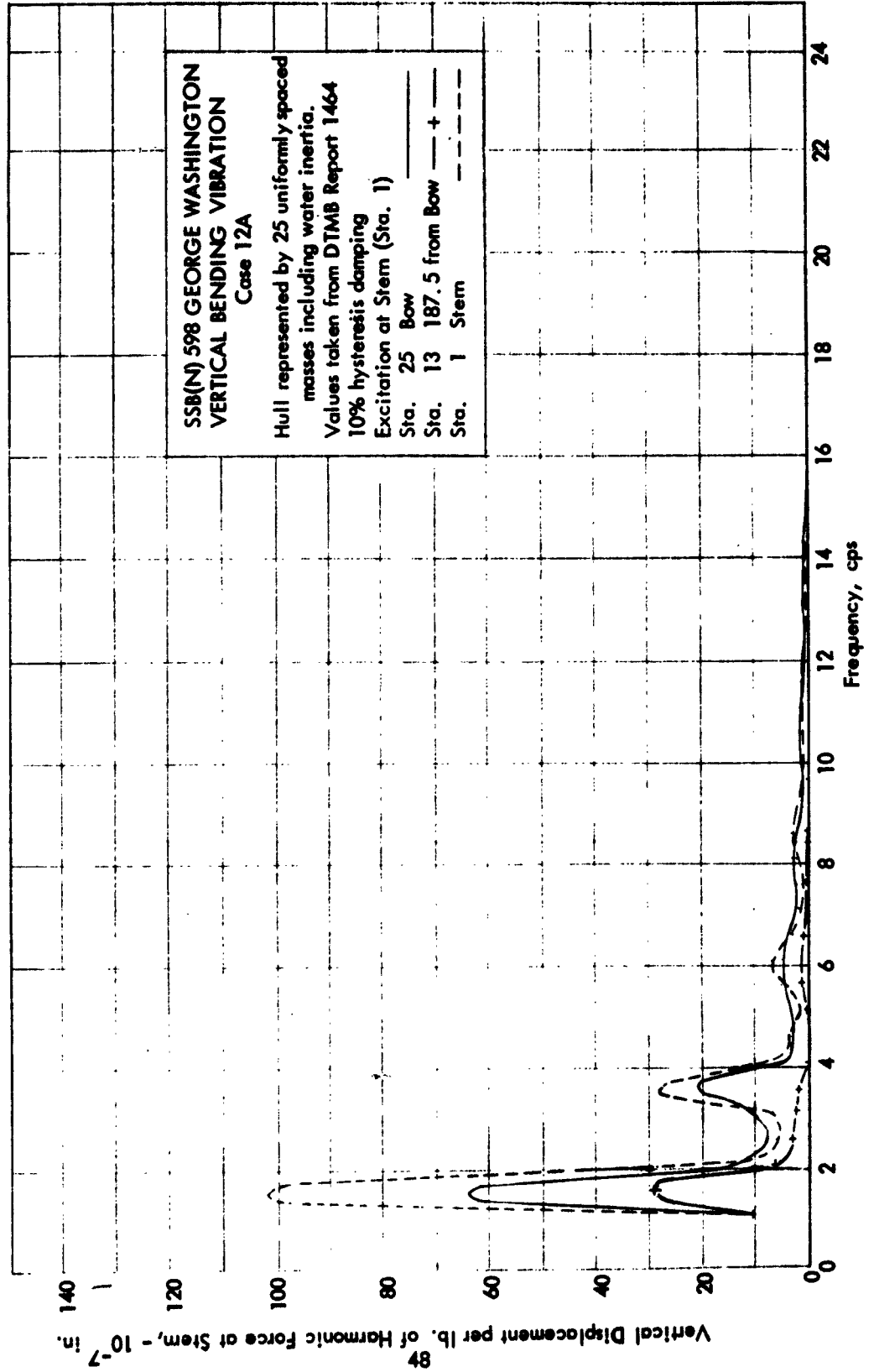


Figure 27 - Displacement in Vertical Bending for 1 lb. Excitation at Stem, Case 12A

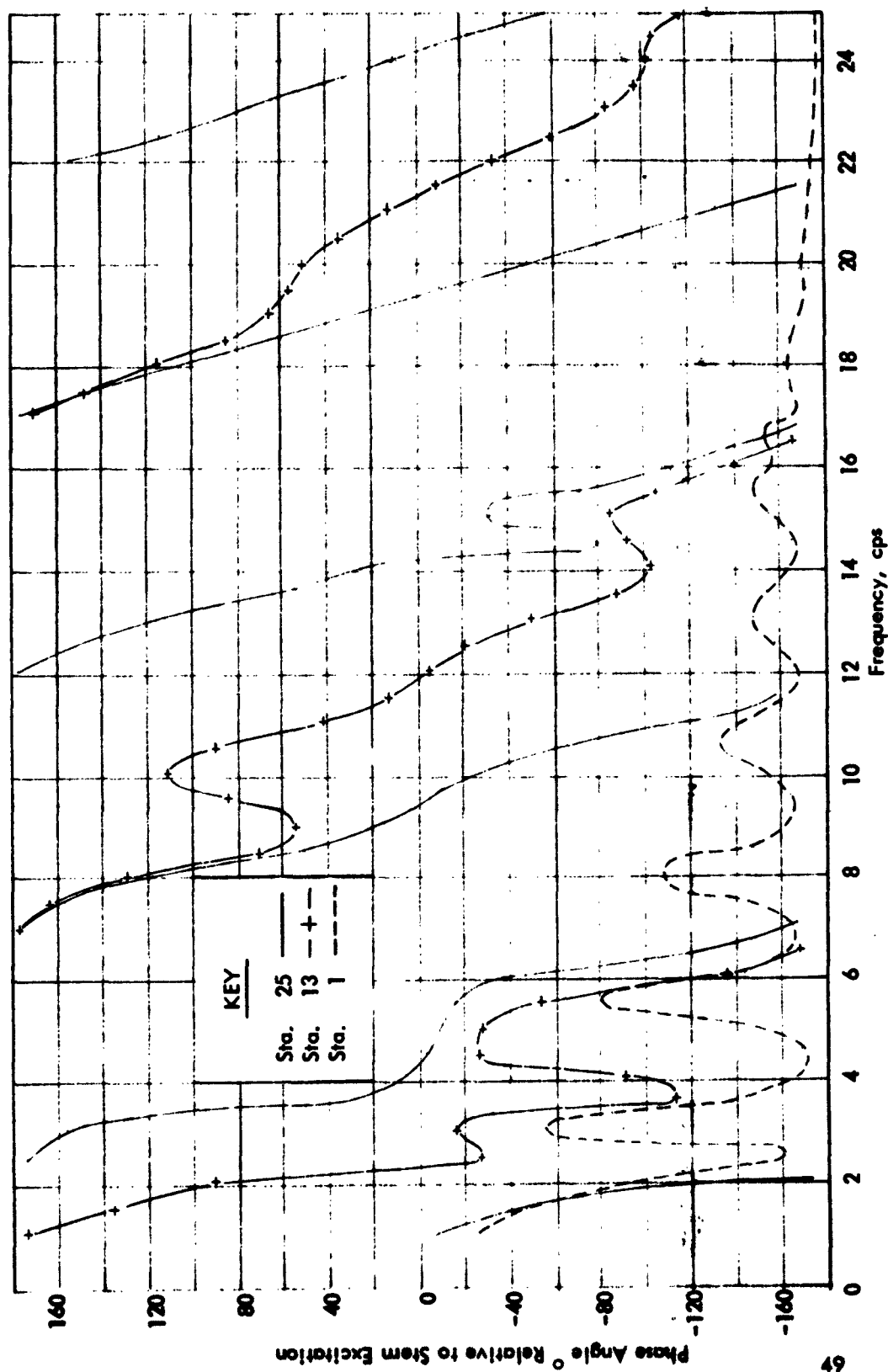


Figure 28—Vertical Bending Phase Angle Relative to Stem Excitation, Case 12A

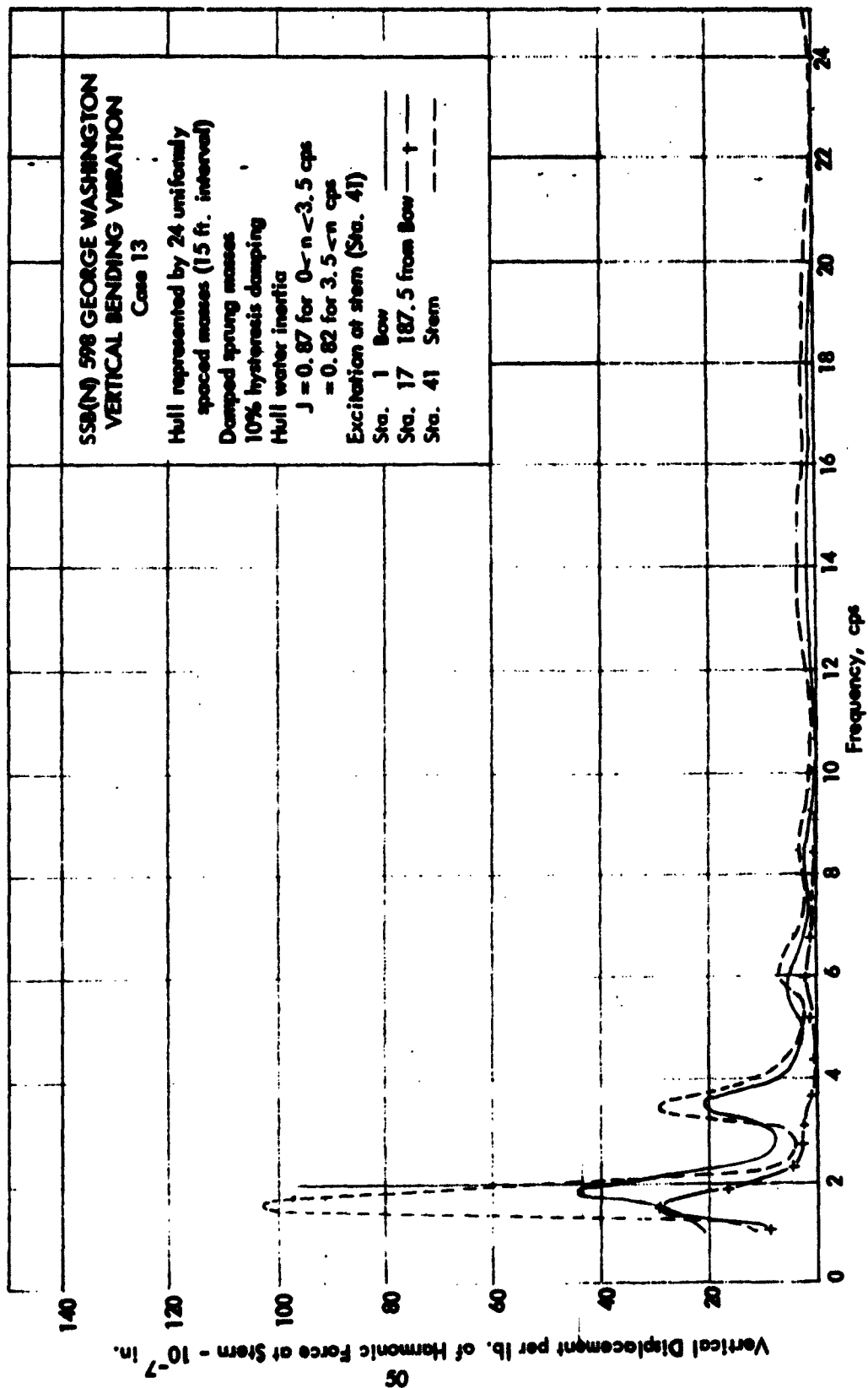


Figure 29—Displacement in Vertical Bending for 1 lb. Excitation at Stern, Case 13

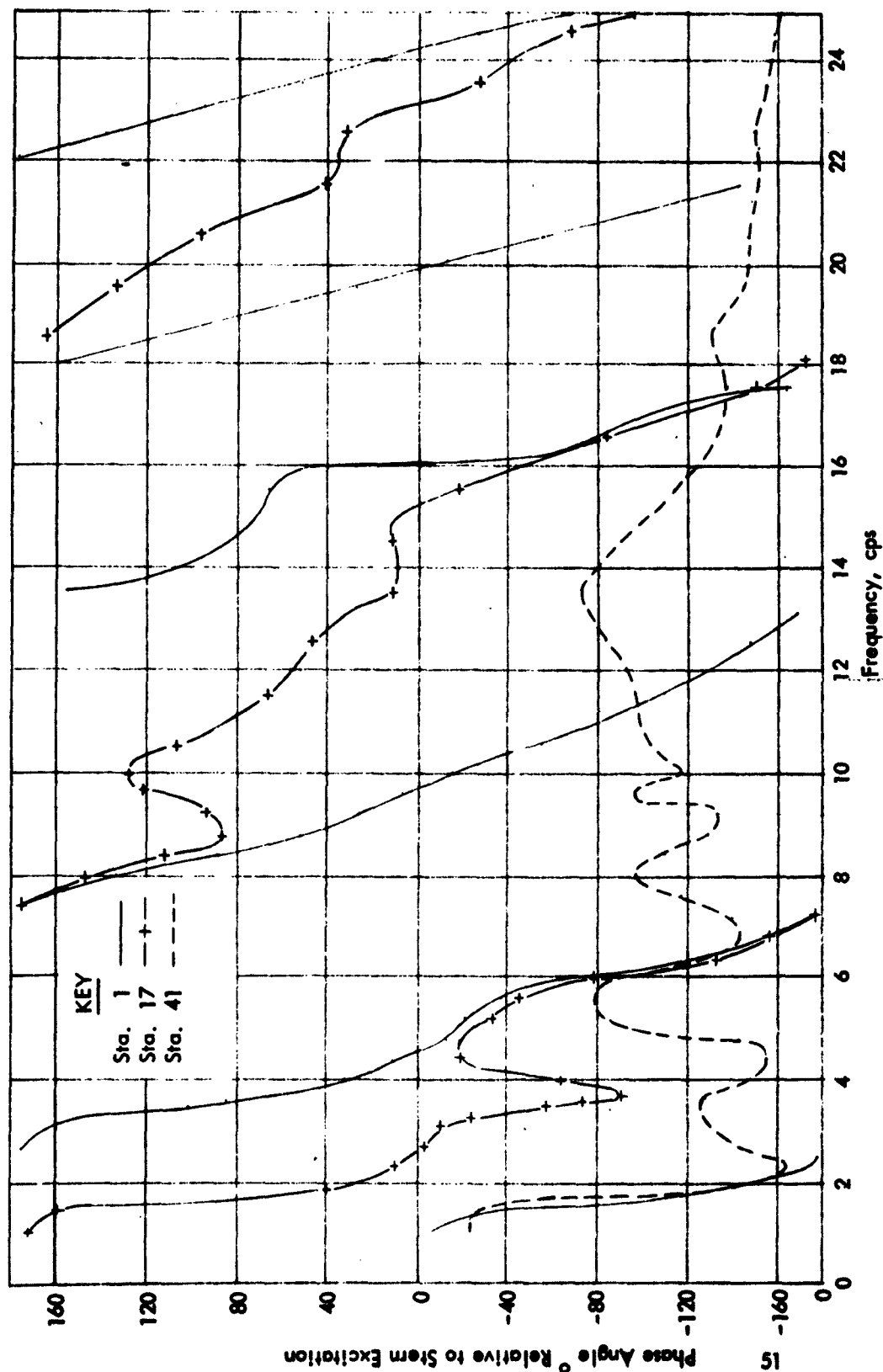


Figure 30 - Vertical Bending Phase Angle Relative to Stem Excitation, Case 13

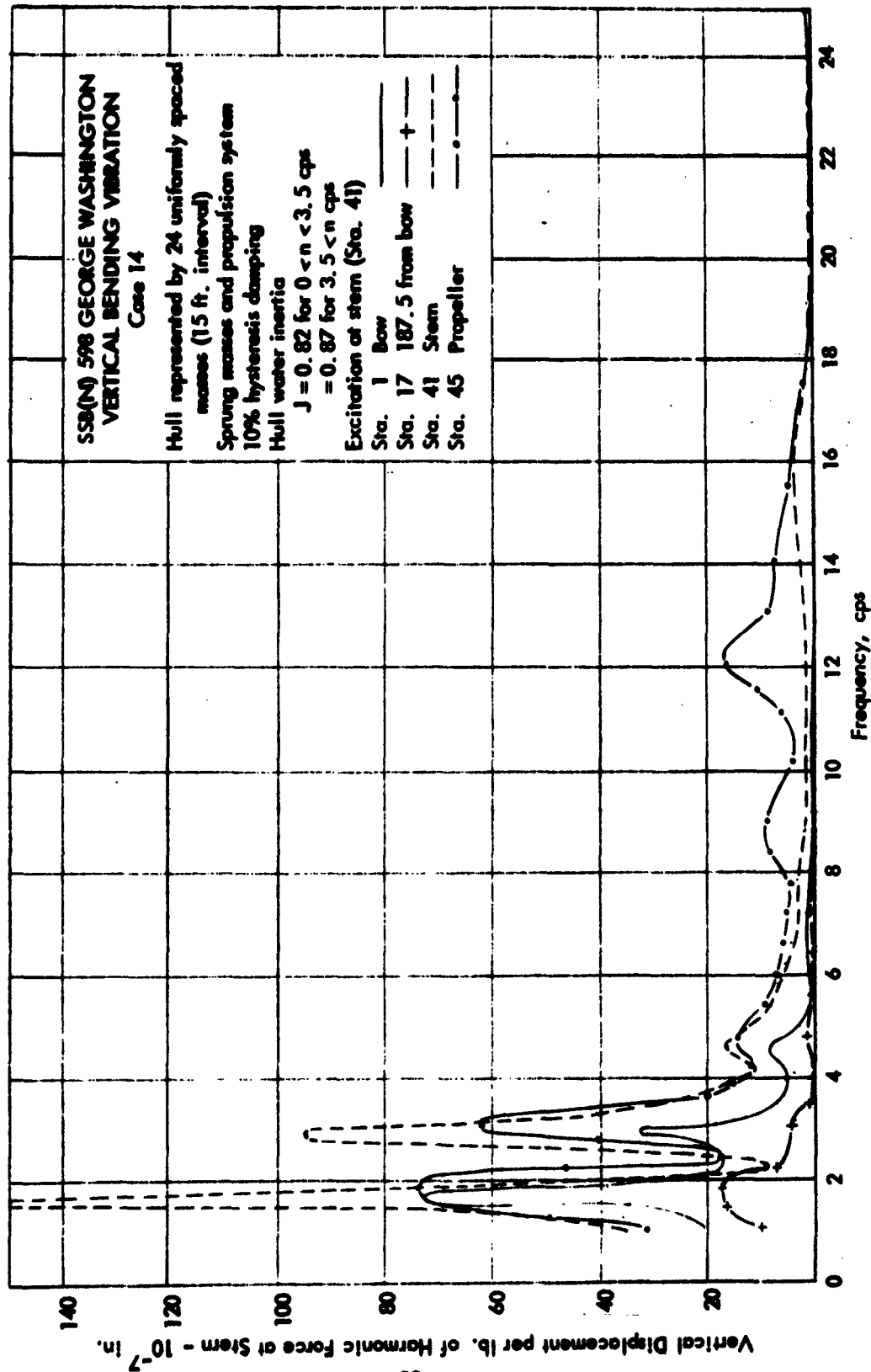


Figure 31 Displacement in Vertical Bending for 1 lb. Excitation at Stem, Case 14

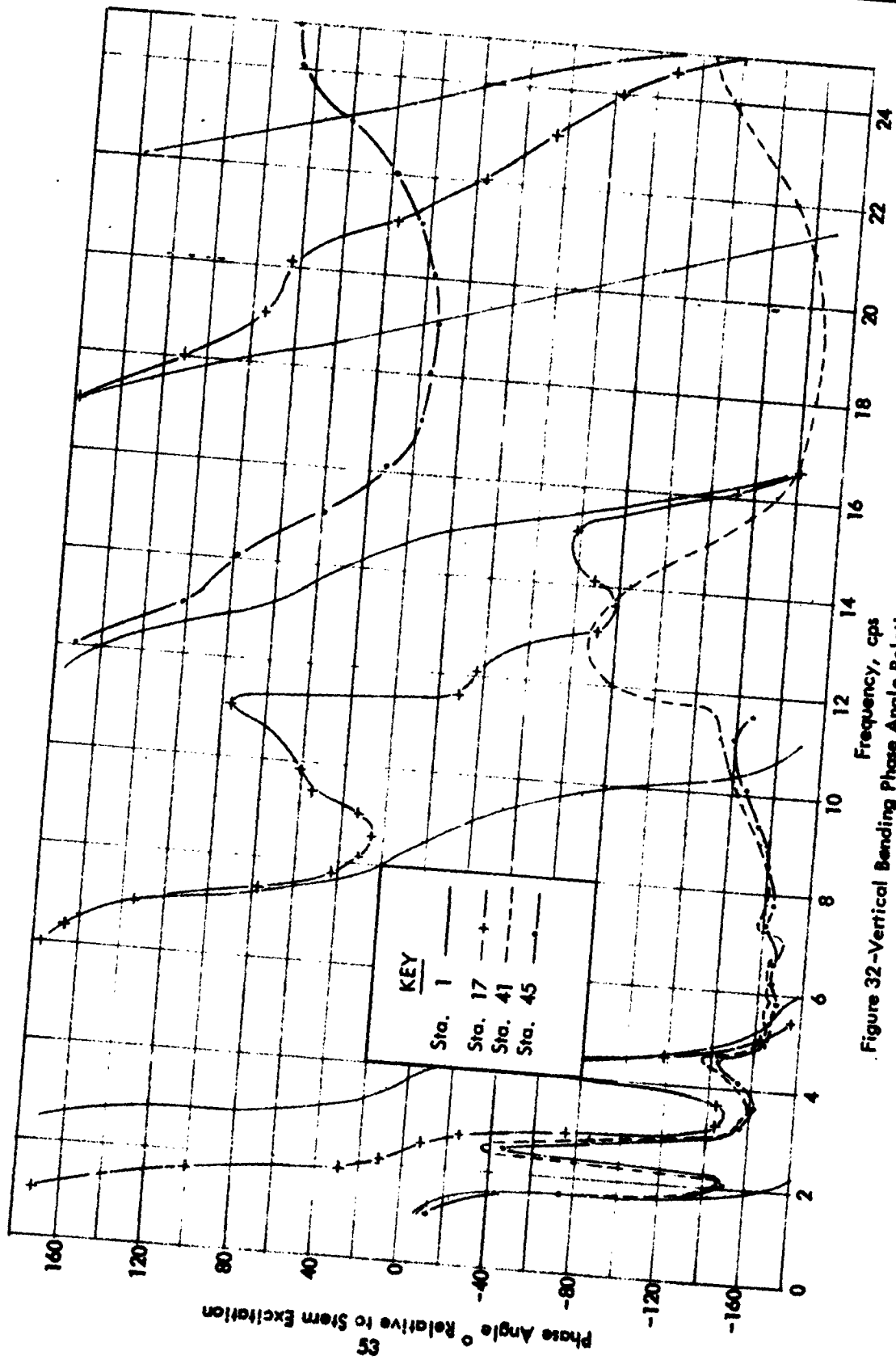


Figure 32 - Vertical Bending Phase Angle Relative to Stem Excitation, Case 14

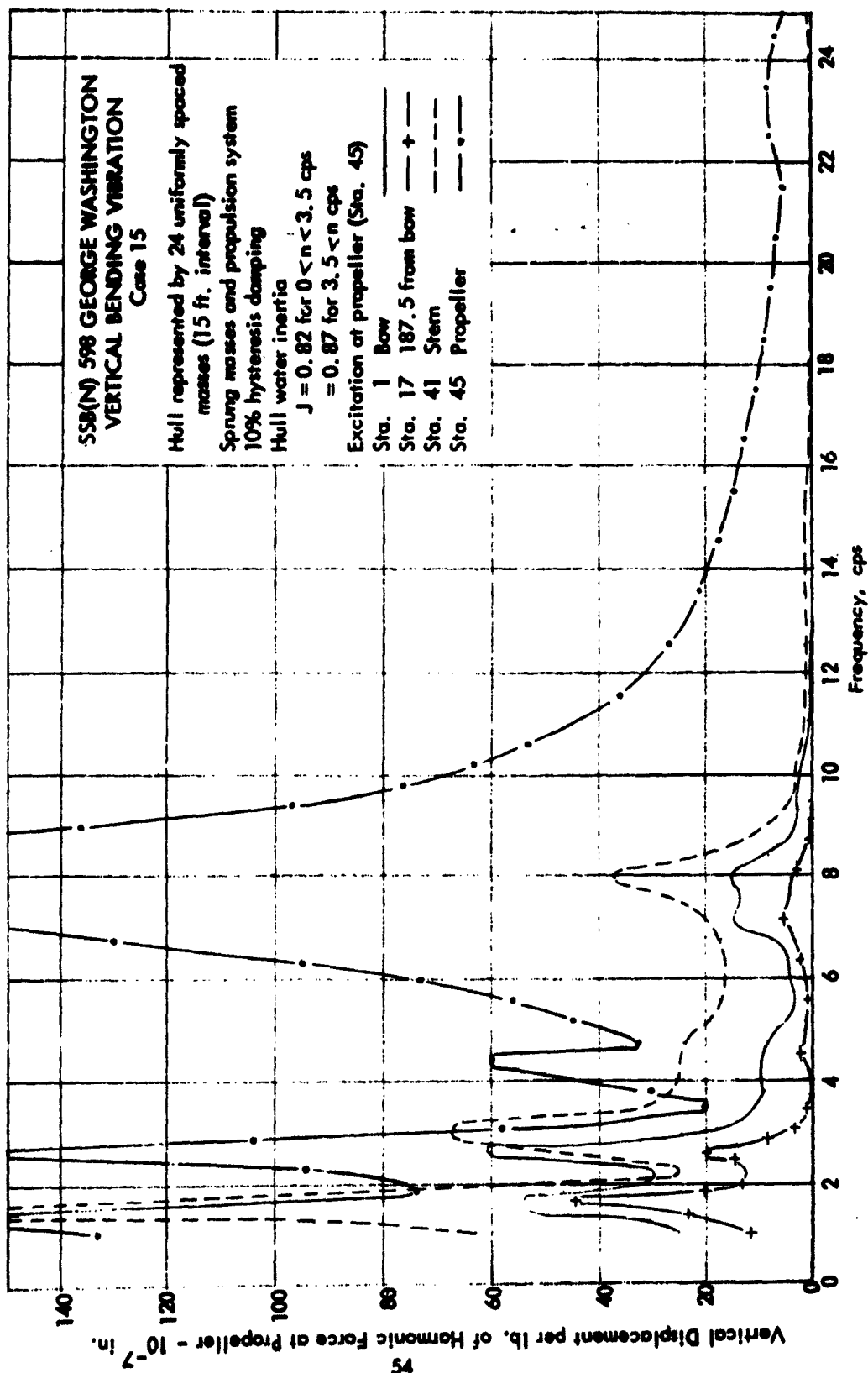


Figure 33 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 15

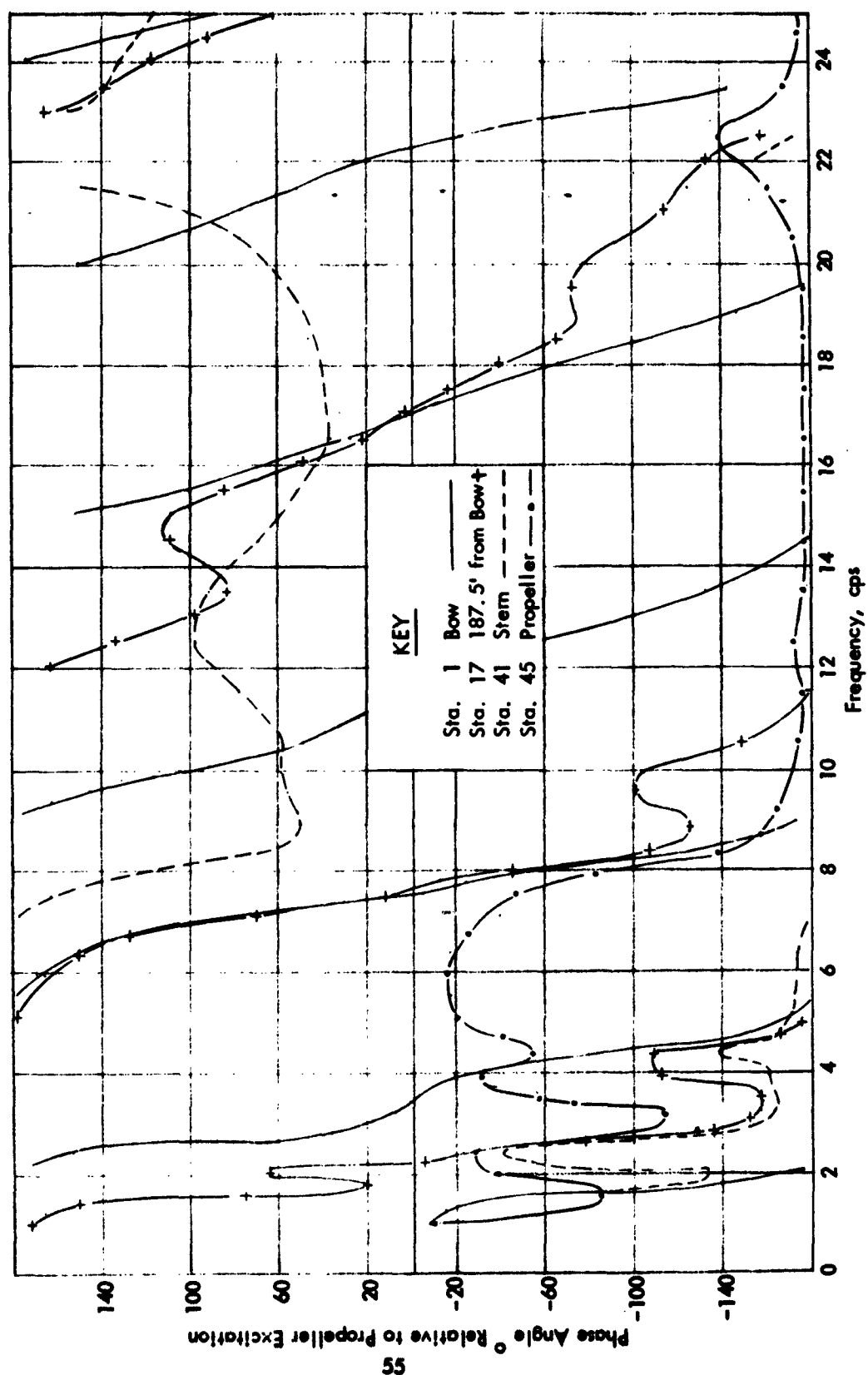


Figure 34 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 15

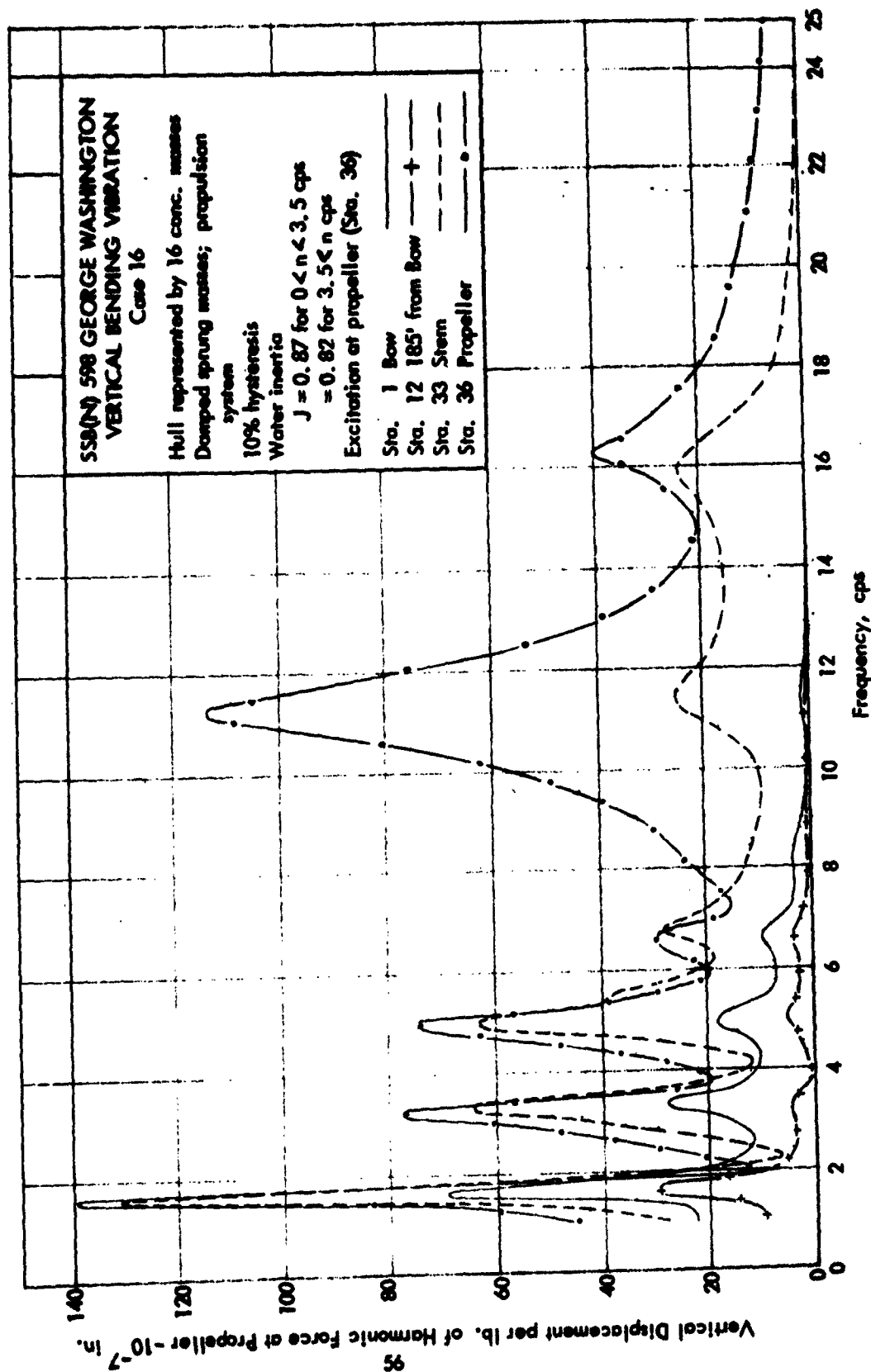


Figure 35-Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 16

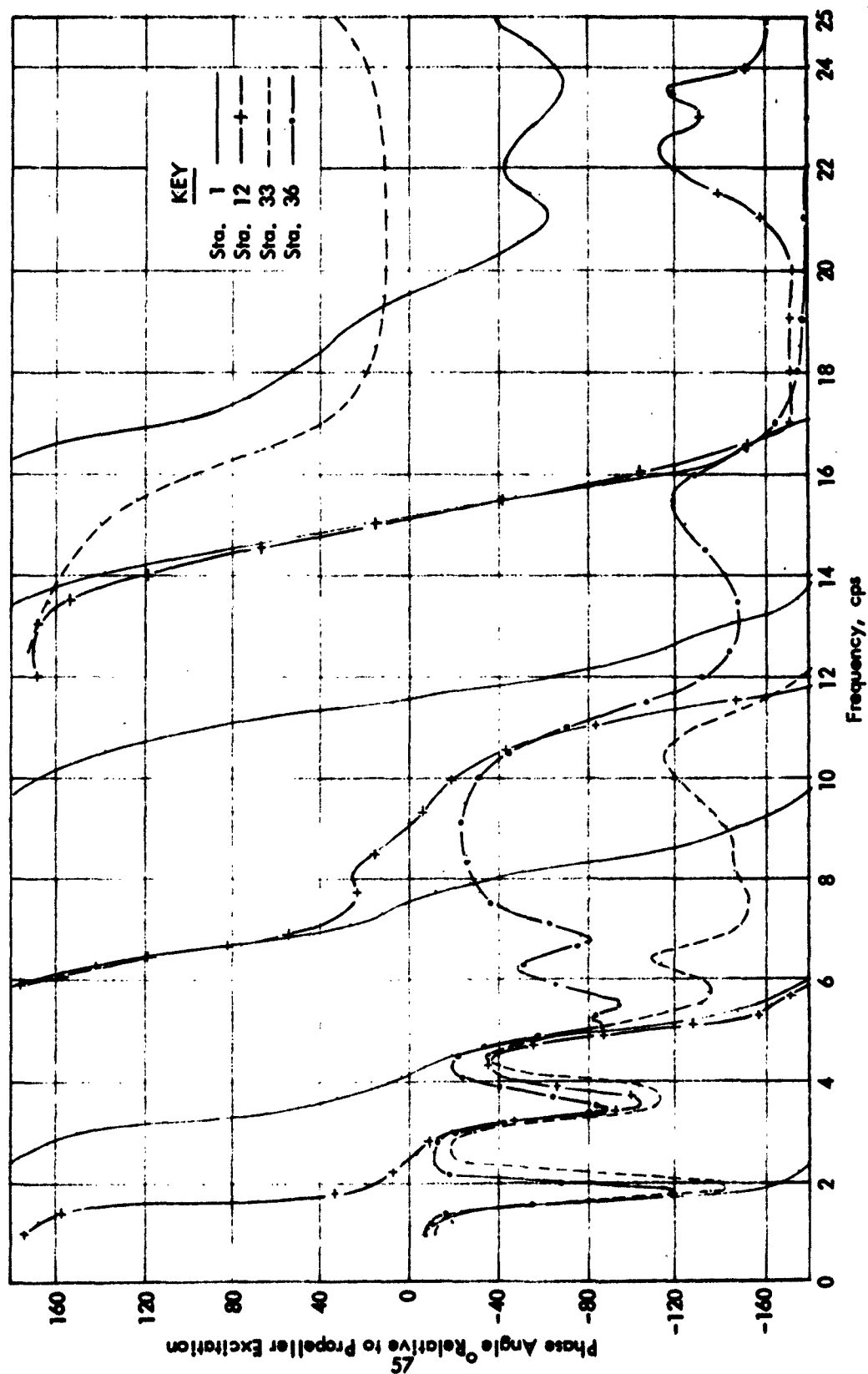


Figure 36-Vertical Bending Phase Angle Relative to Propeller Excitation, Case 16

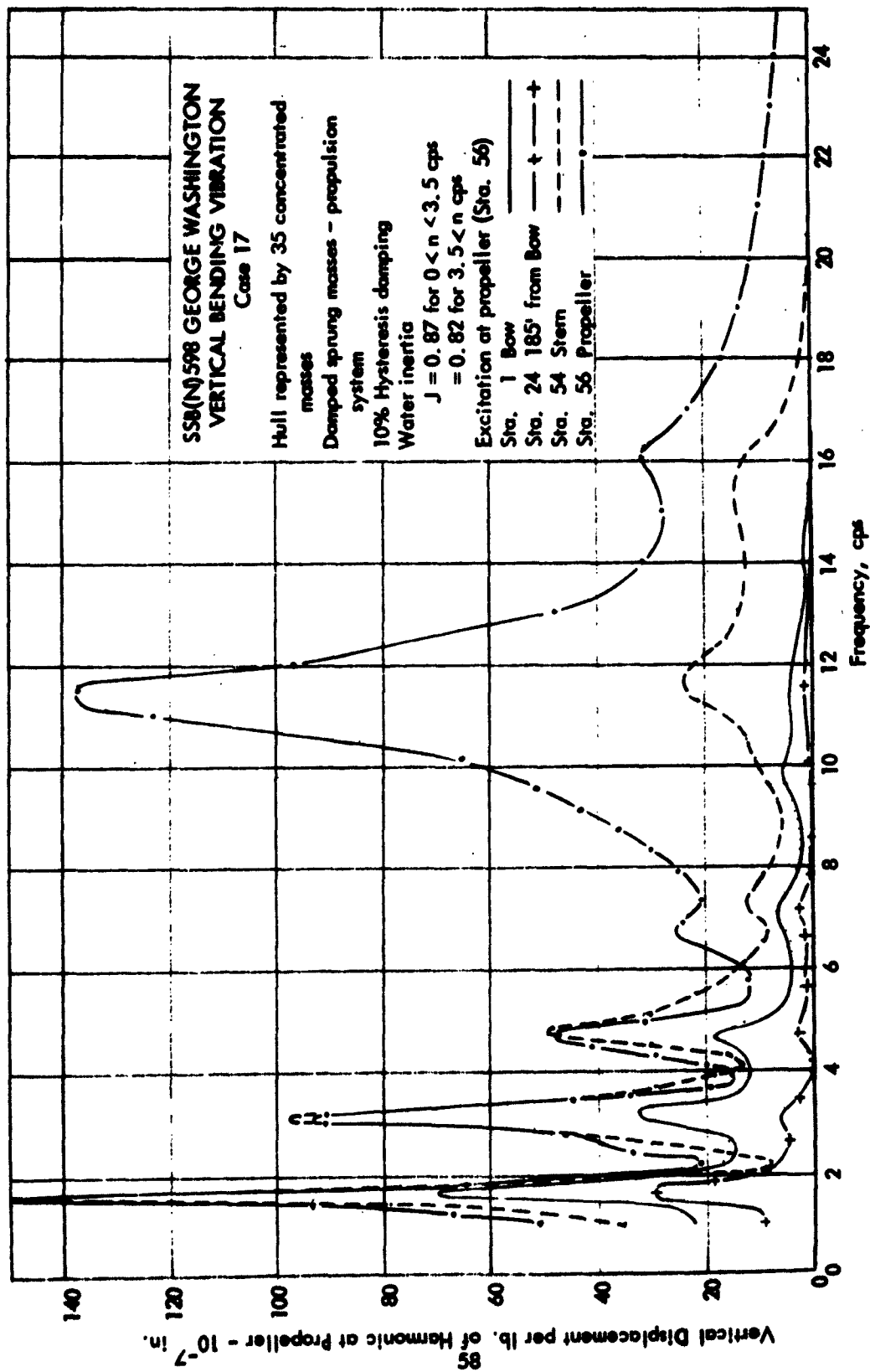


Figure 37-Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 17

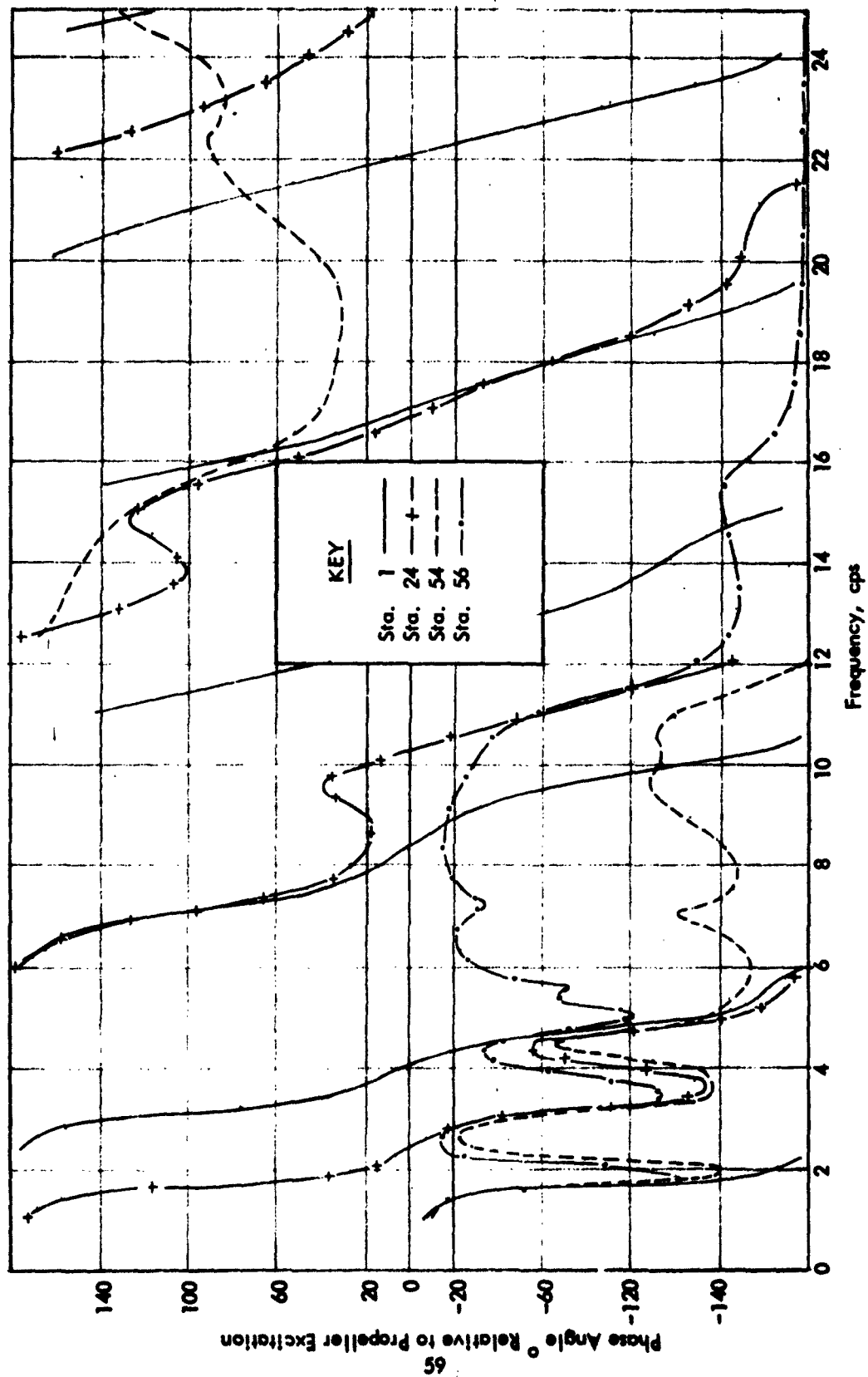


Figure 38 - Vertical Bending Phase Angle Relative to Propeller Excitation, Case 17

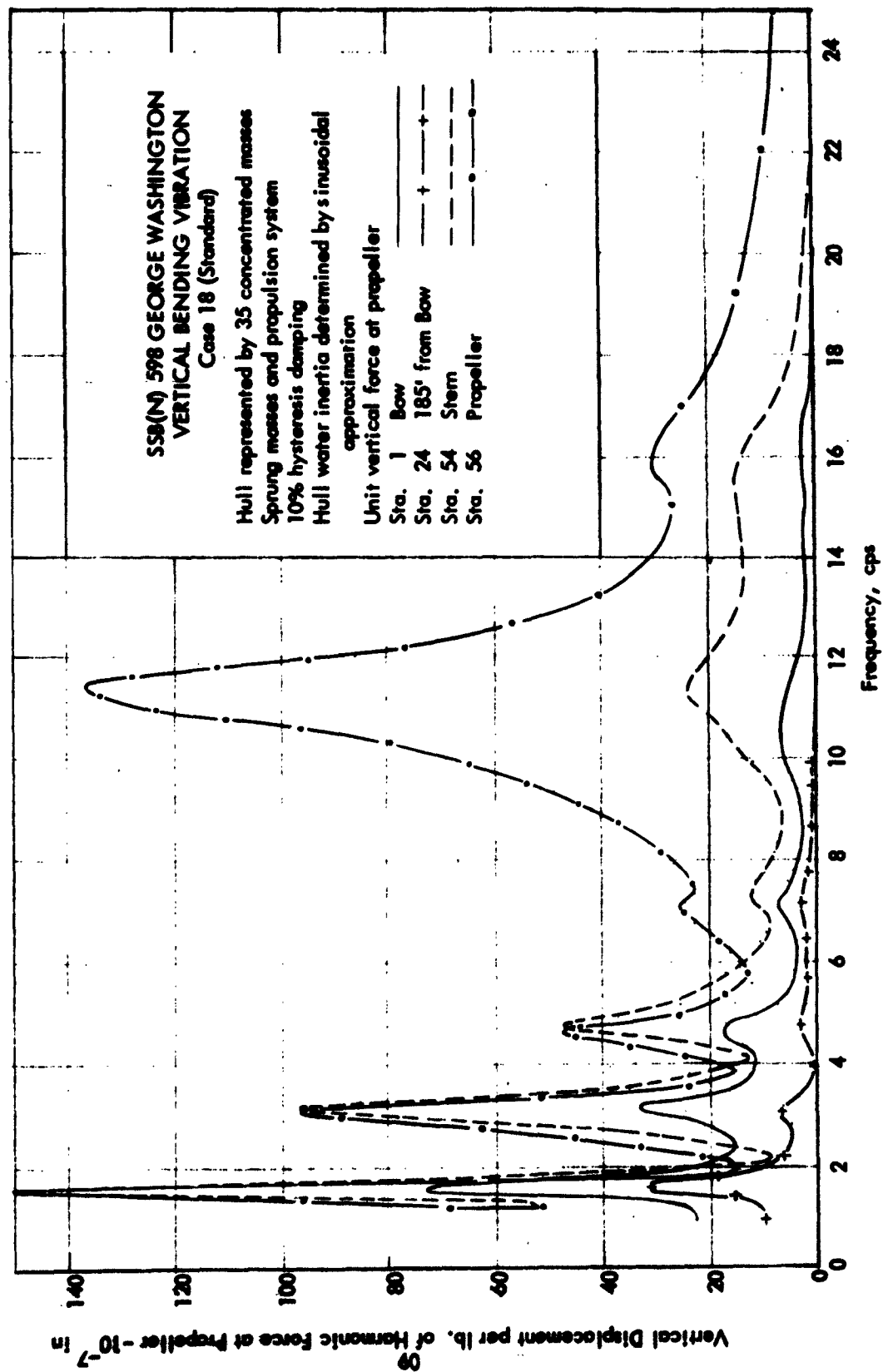


Figure 39 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 18 (Standard)

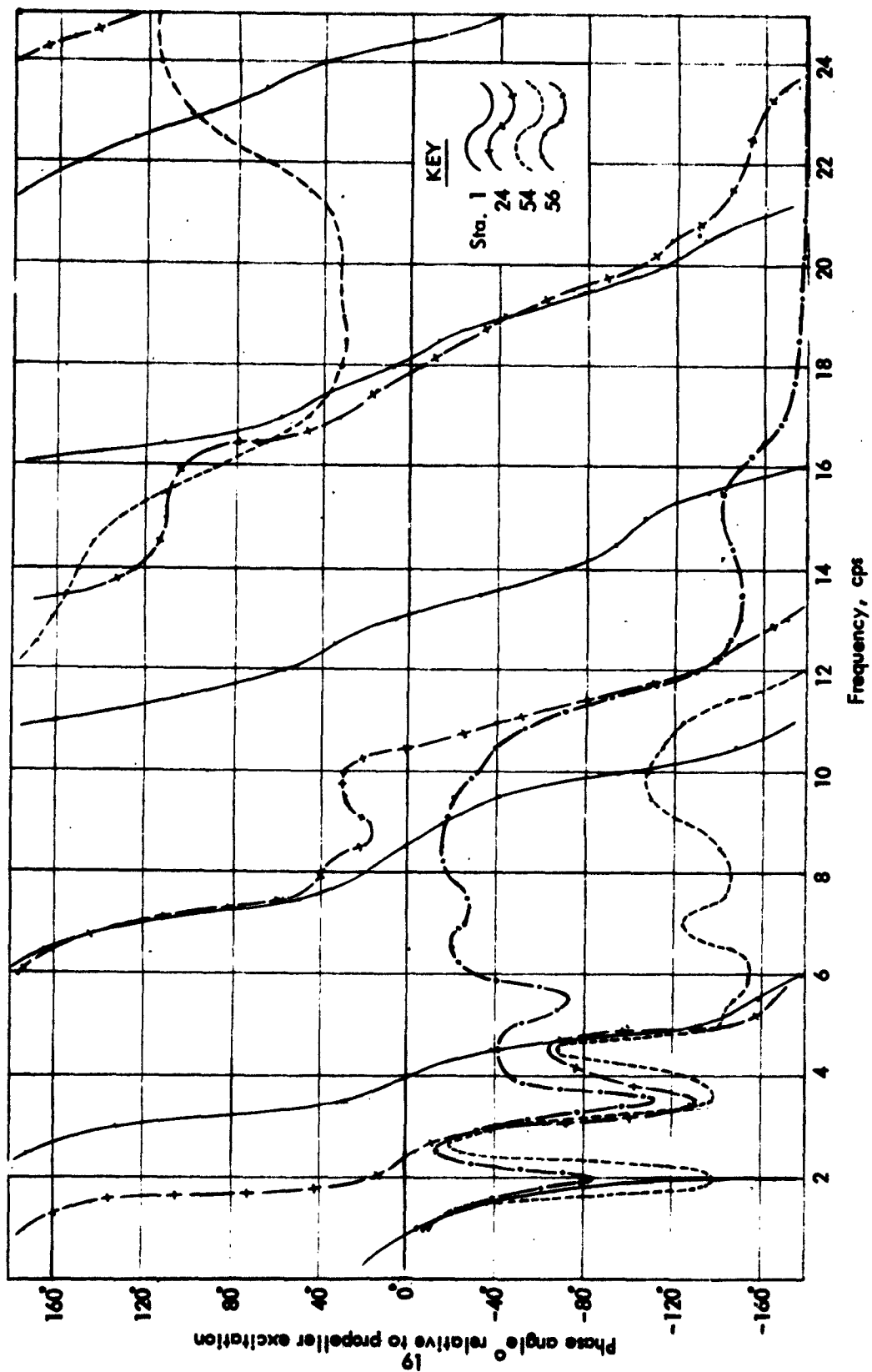


Figure 40 - Vertical Bending Phase Angle Relative to Propeller Excitation, Case 18 (Standard)

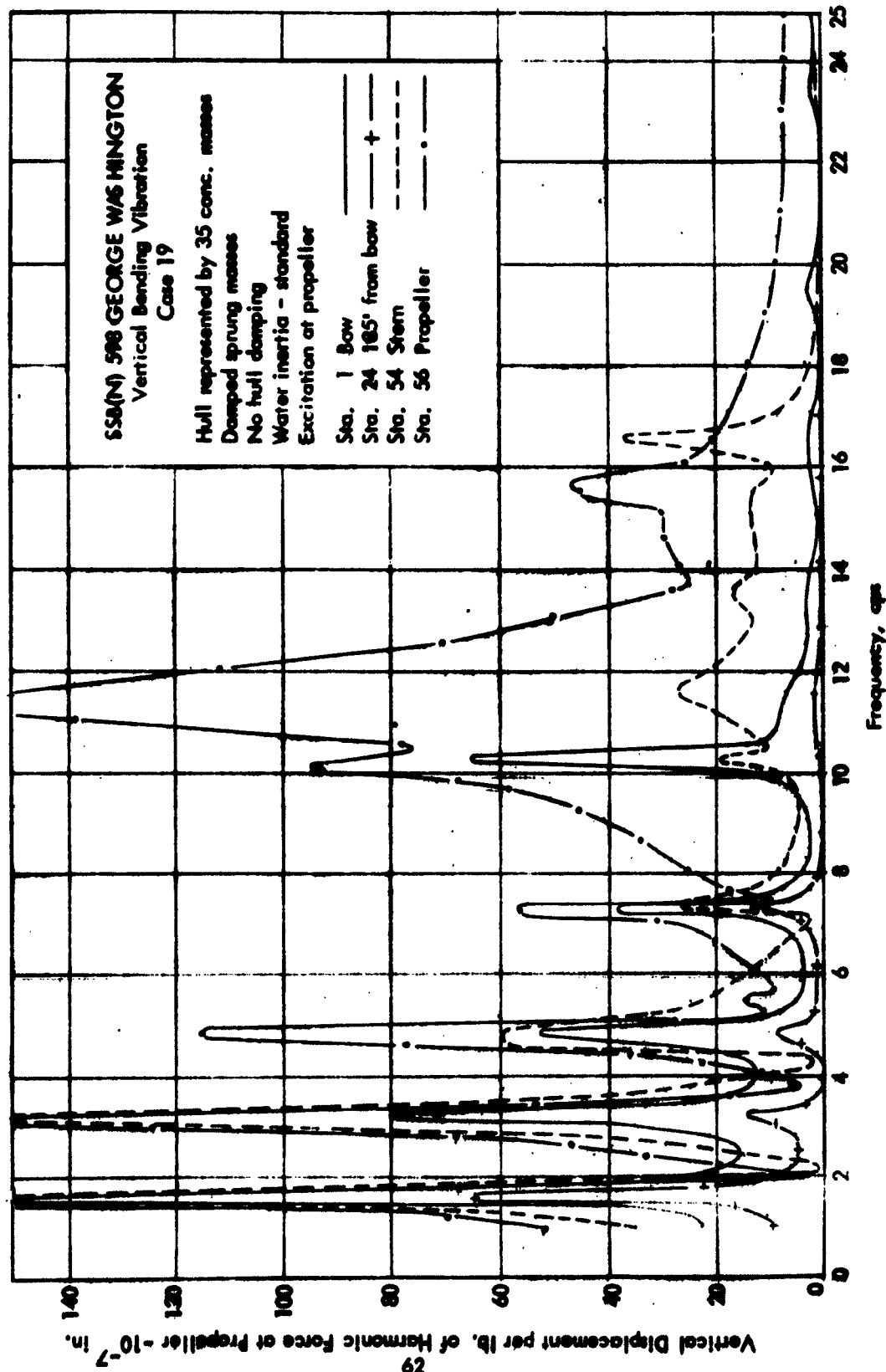


Figure 41-Displacement in Vertical Bending for 1 lb. Force of Excitation at Propeller, Case 19

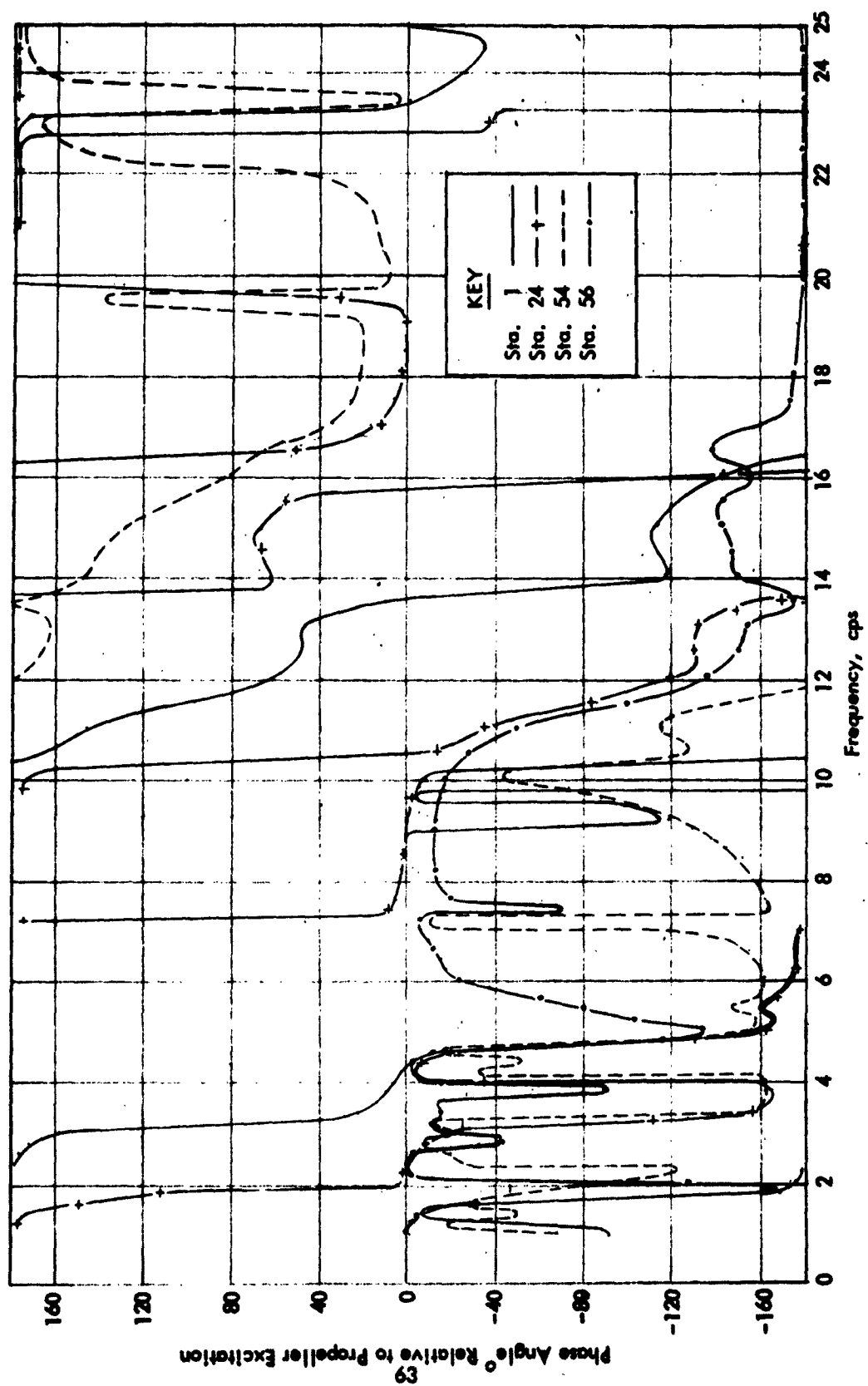


Figure 42 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 19

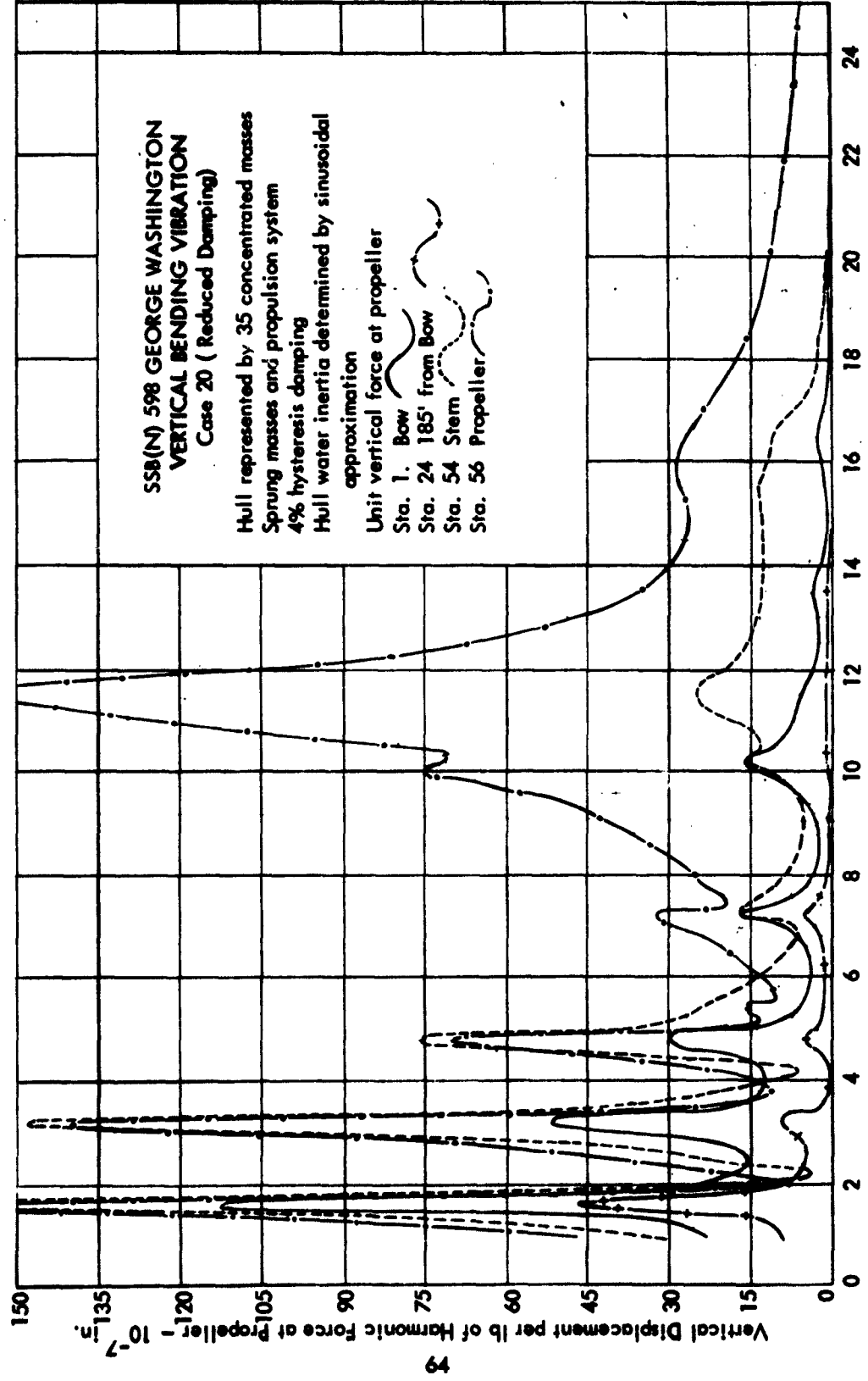


Figure 43- Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 20 (Reduced Damping)

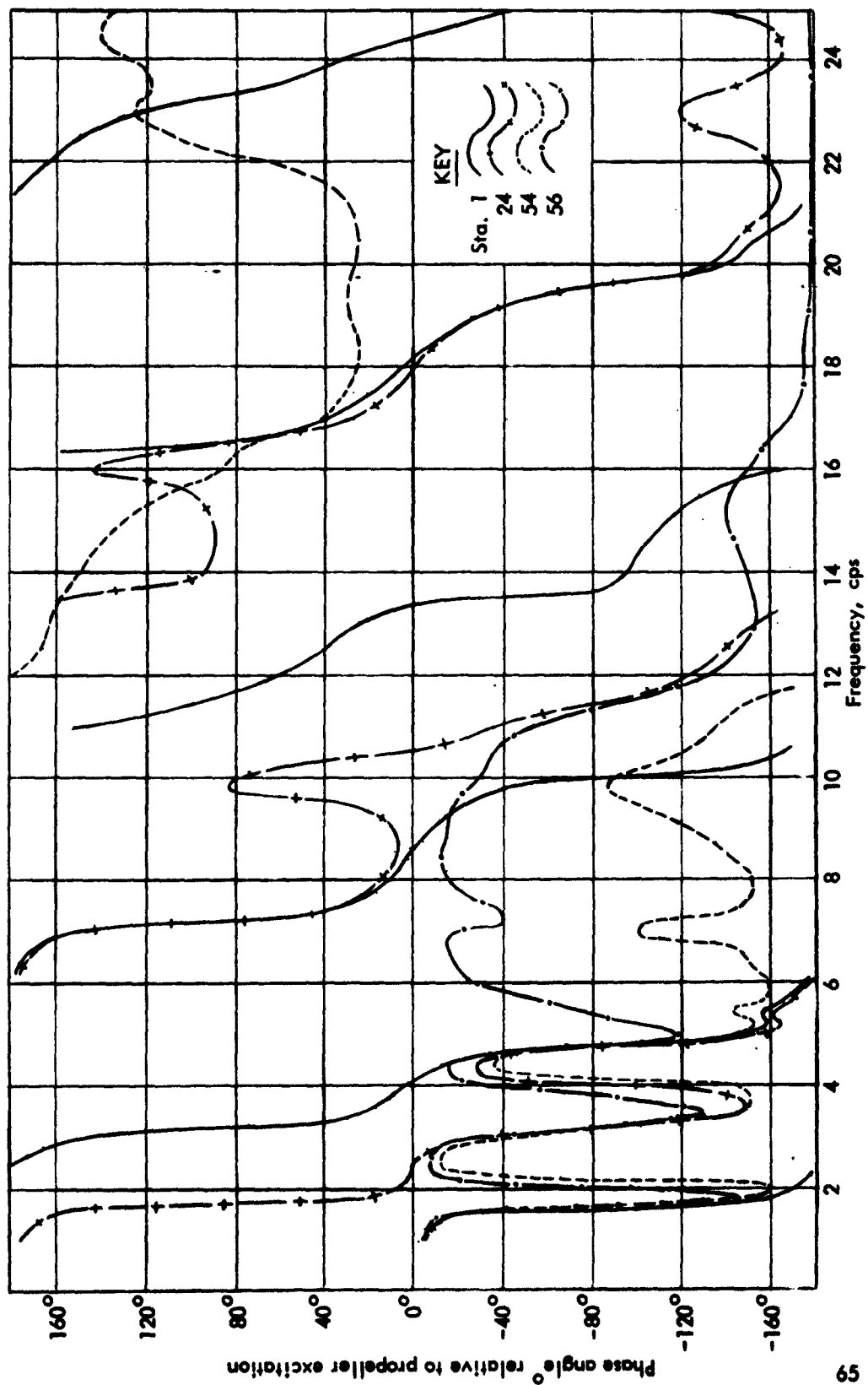


Figure 44-Vertical Bending Phase Angle Relative to Propeller Excitation, Case 20 (Reduced Damping)

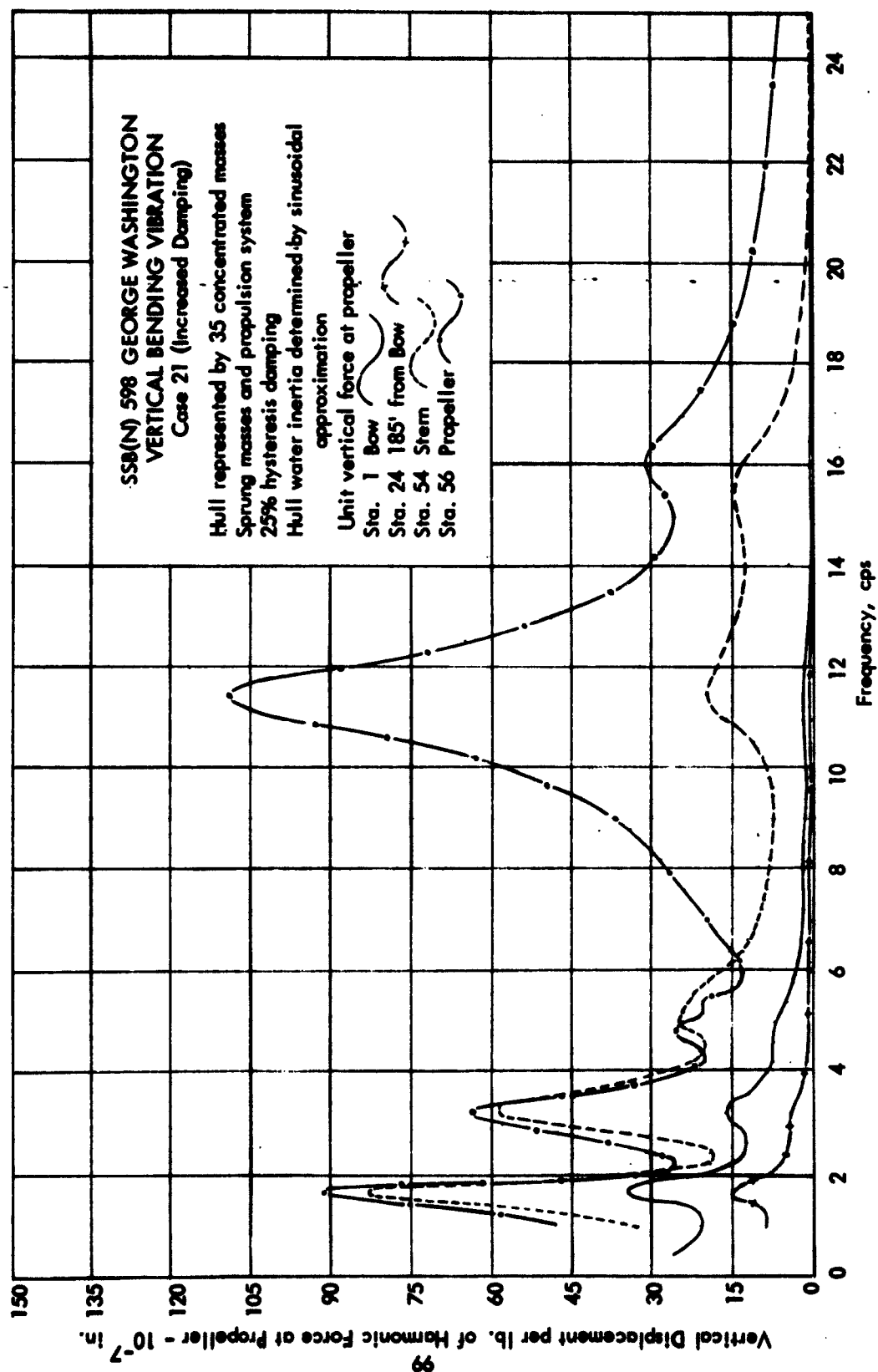


Figure 45 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 21 (Increased Damping)

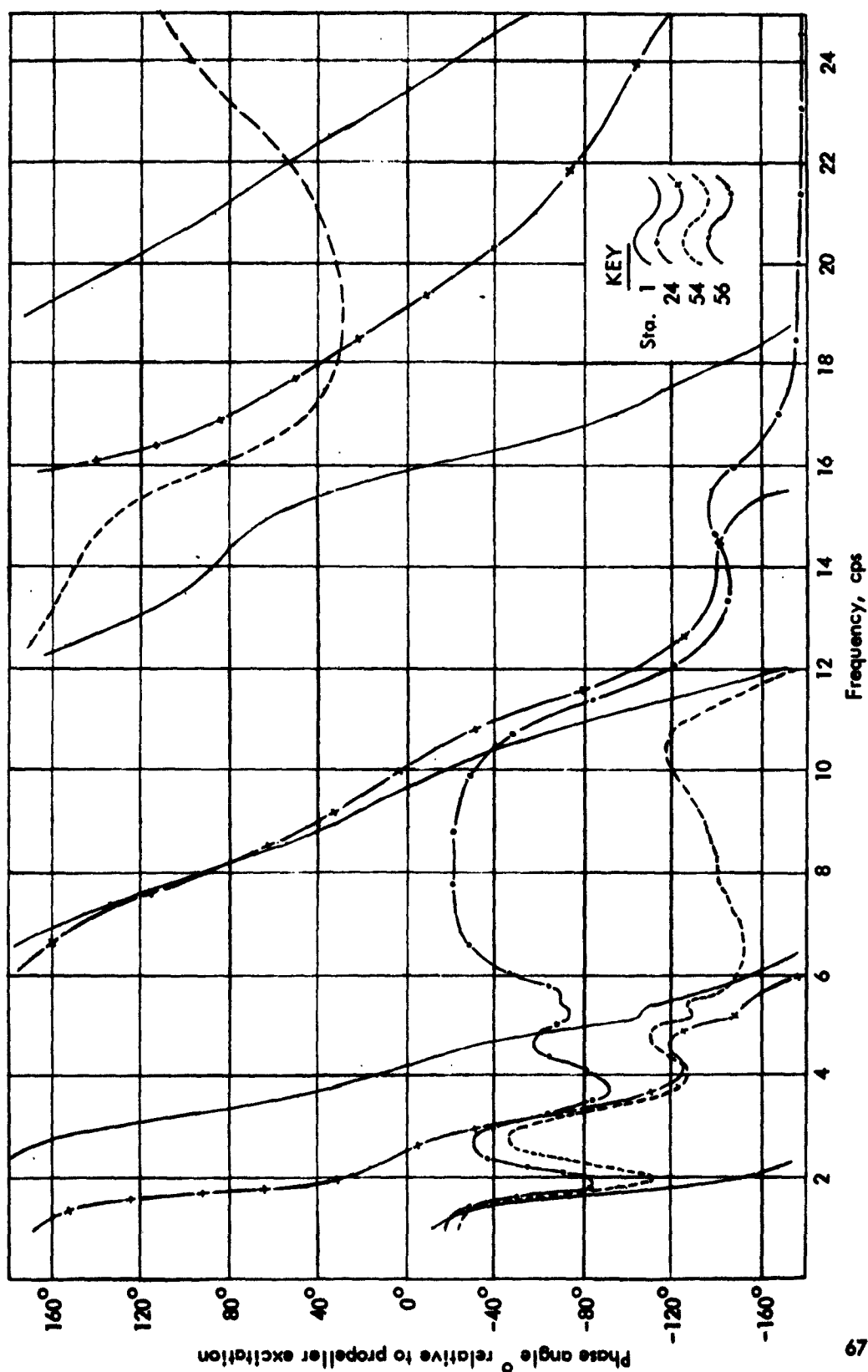


Figure 46 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 21 (Increased Damping)

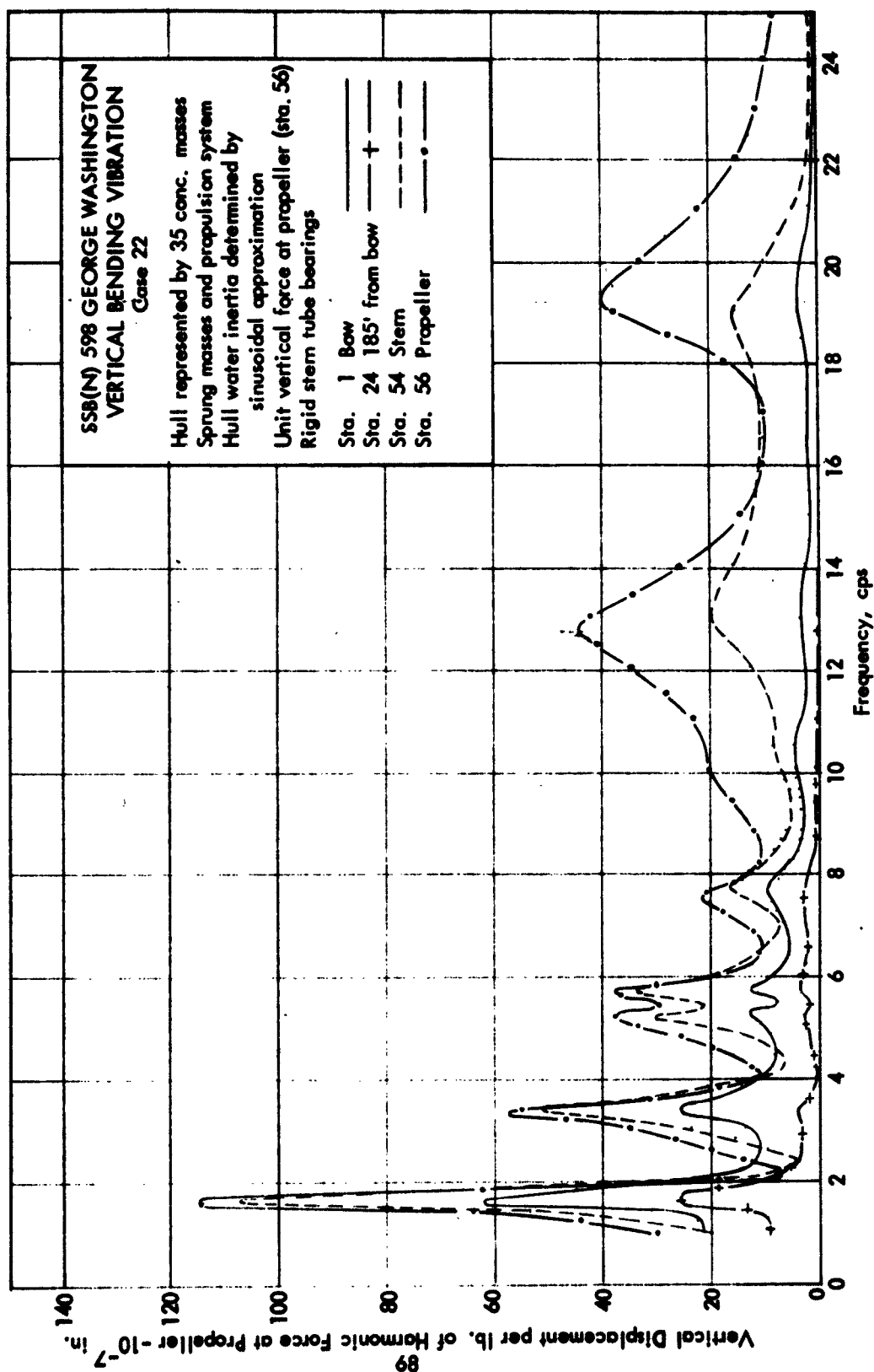


Figure 47-Displacement in Vertical Bending for 1 lb. excitation at Propeller, Case 22

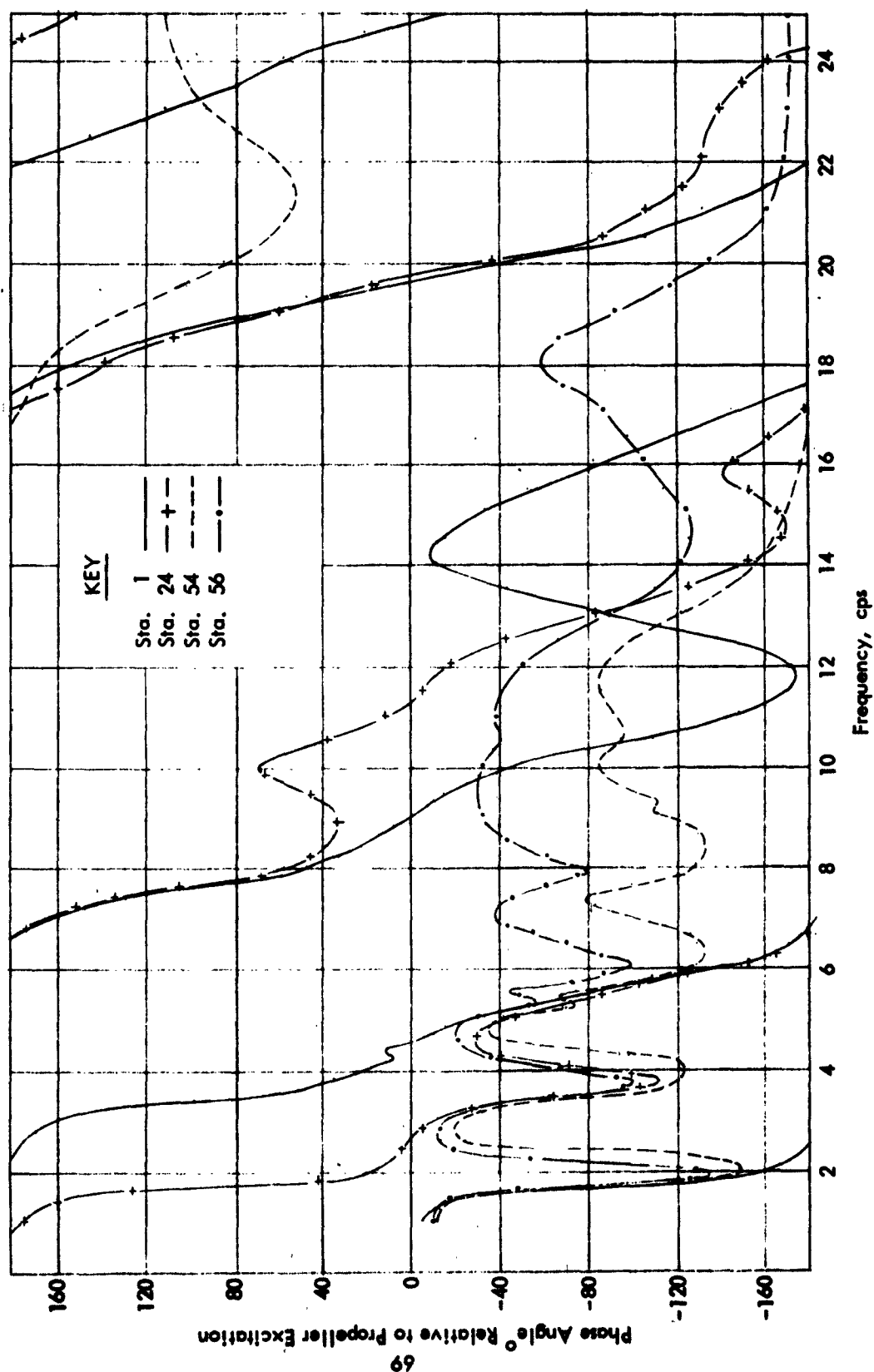


Figure 48 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 22

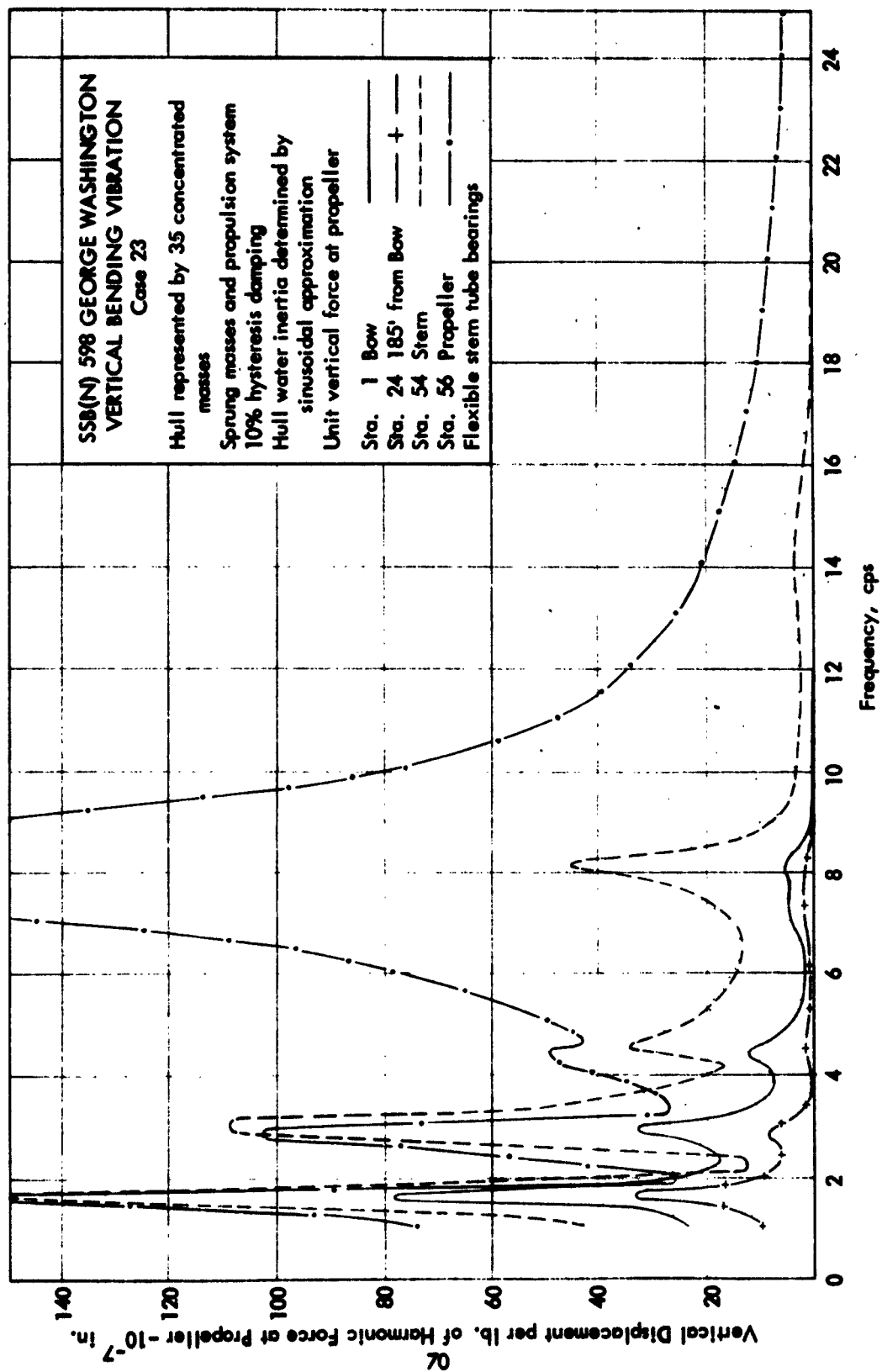


Figure 49-Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 23

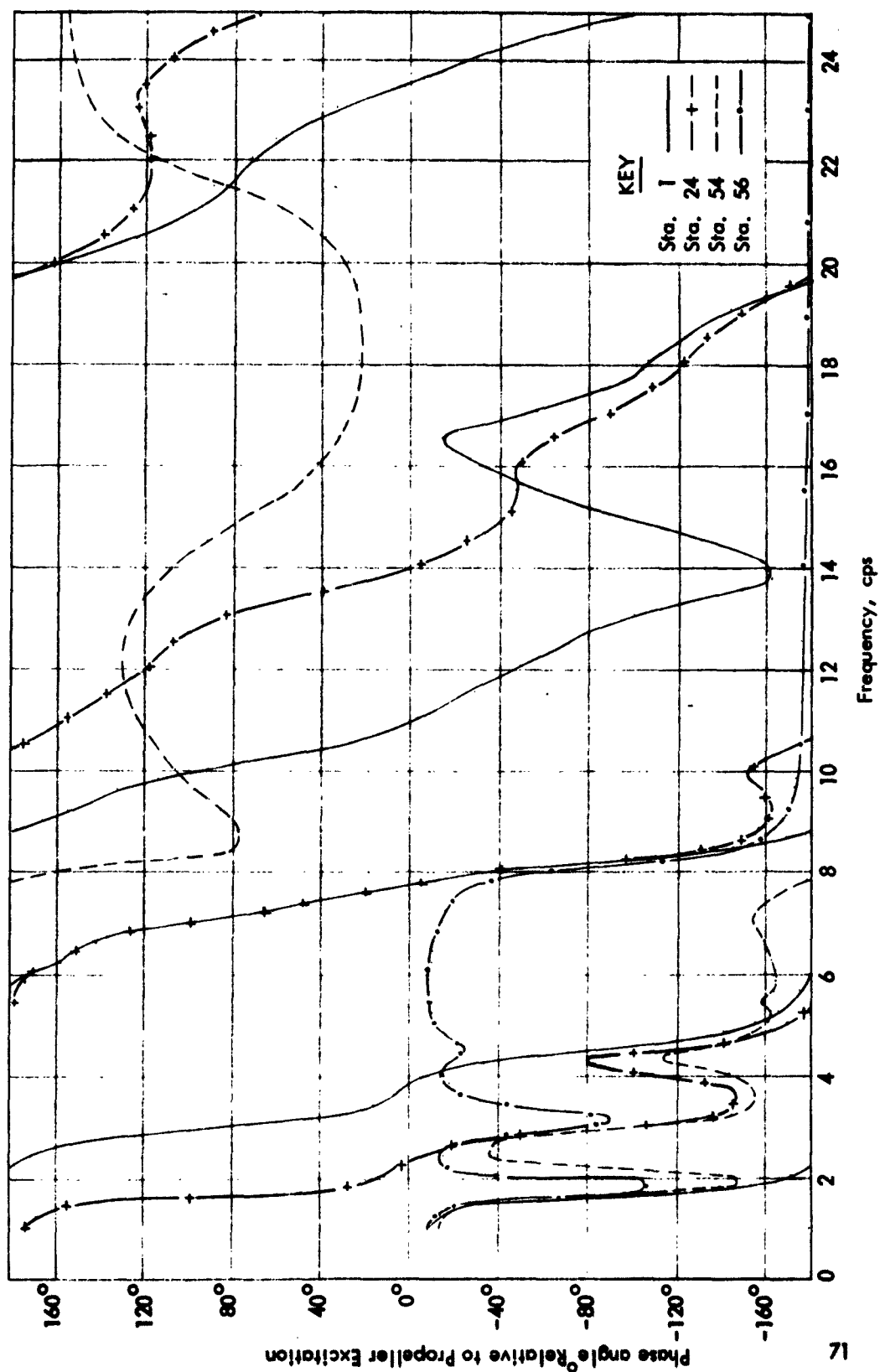


Figure 50-Vertical Bending Phase Angle Relative to Propeller Excitation, Case 23

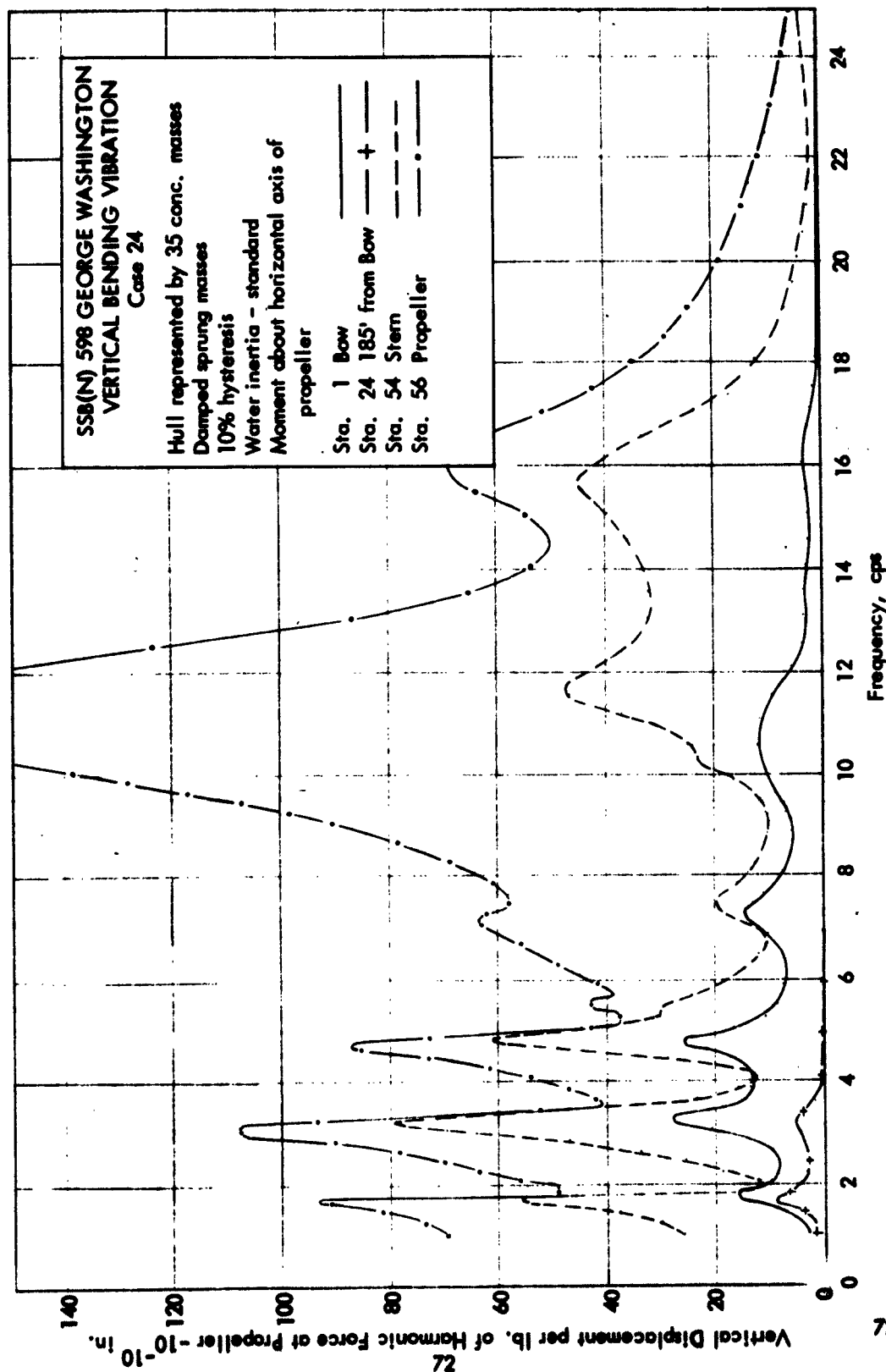


Figure 51 - Displacement in Inertial Bending for 1 lb. Excitation at Propeller, Case 24

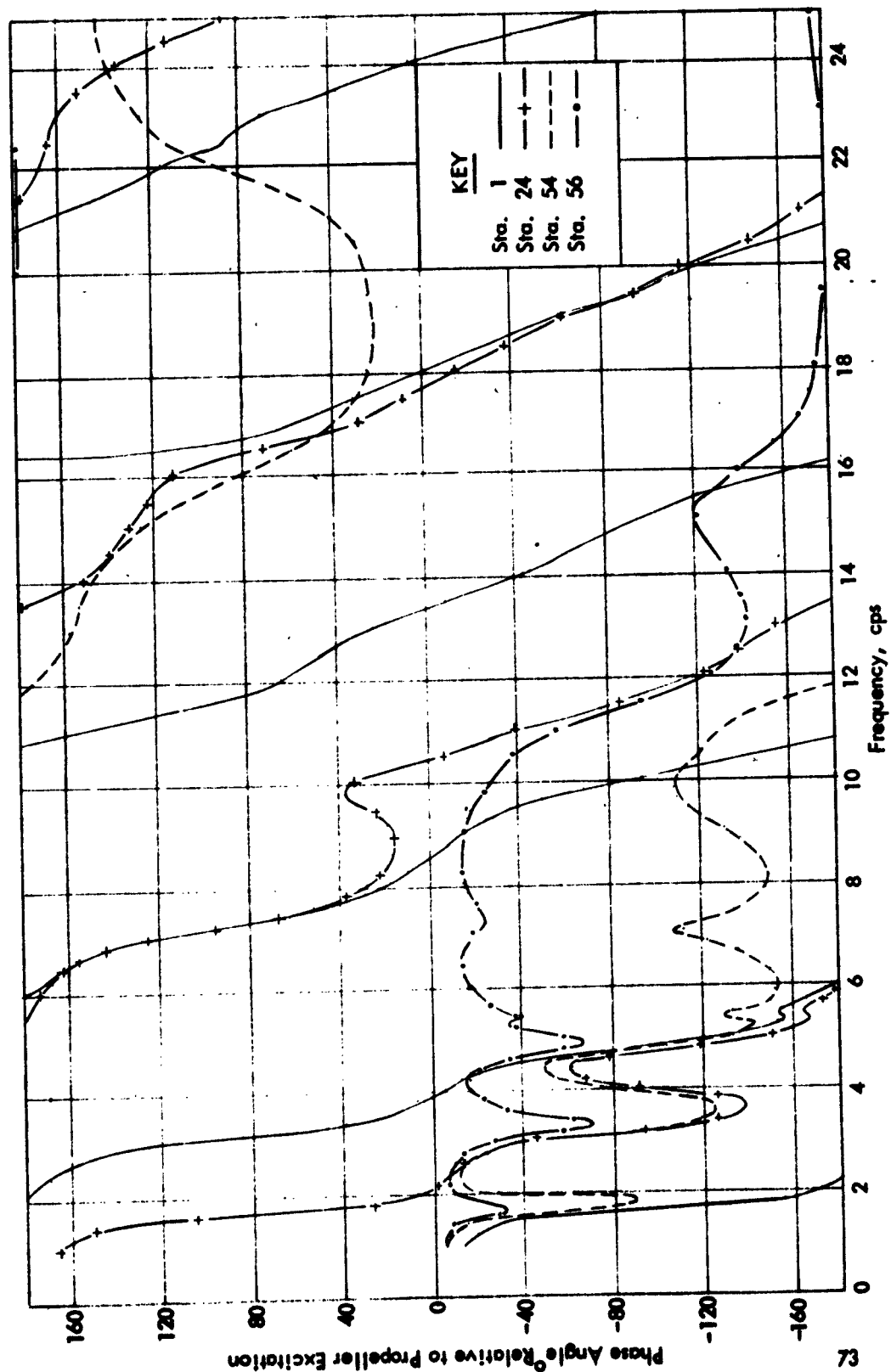


Figure 52 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 24

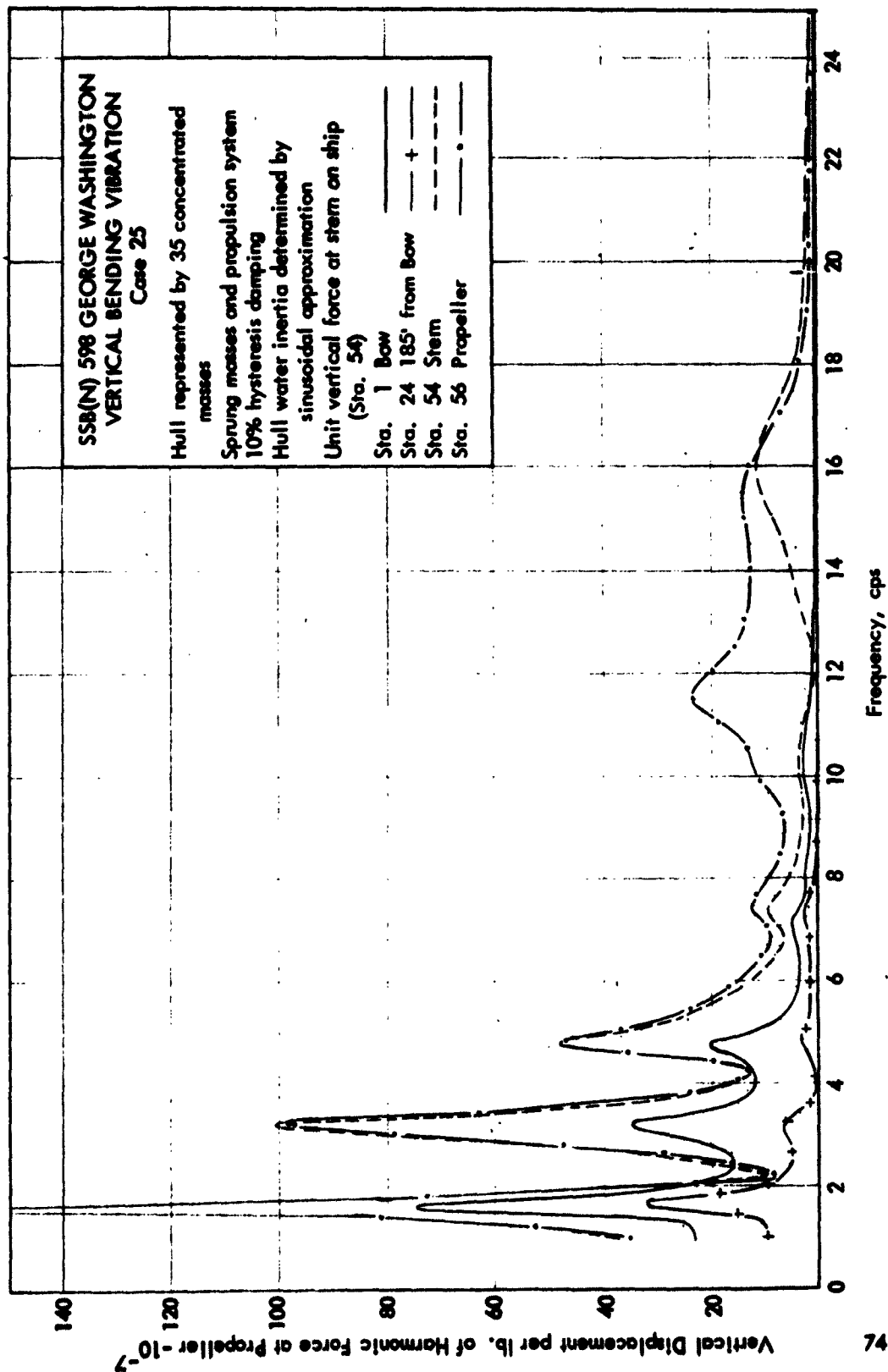


Figure 53-Displacement in Vertical Bending for 1 lb. excitation on hull, Case 25

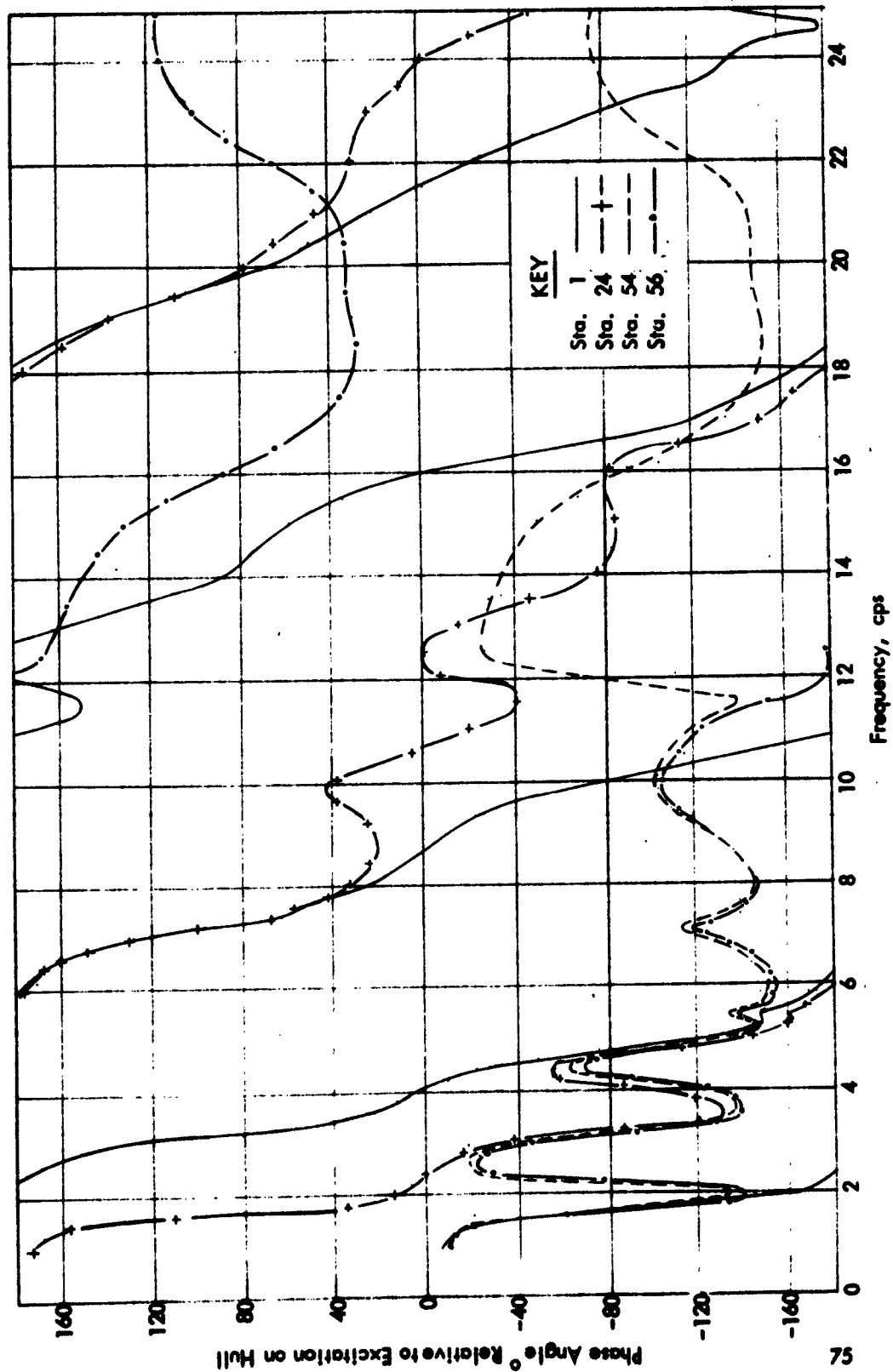


Figure 54 - Vertical Bending Phase Angle Relative to Excitation on Hull, Case 25

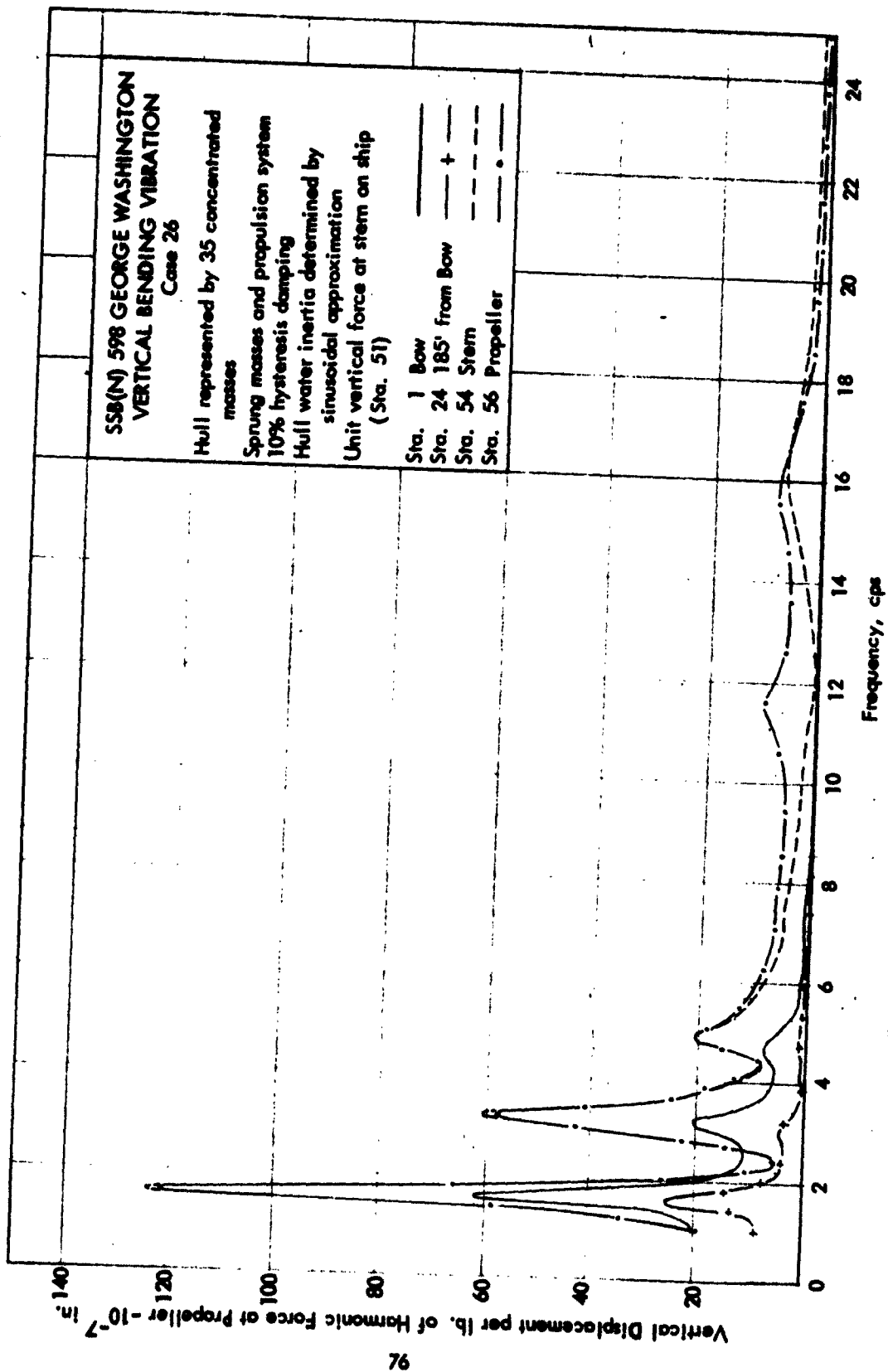


Figure 55 - Displacement in Vertical Bending for 1 lb. excitation on hull, Case 26

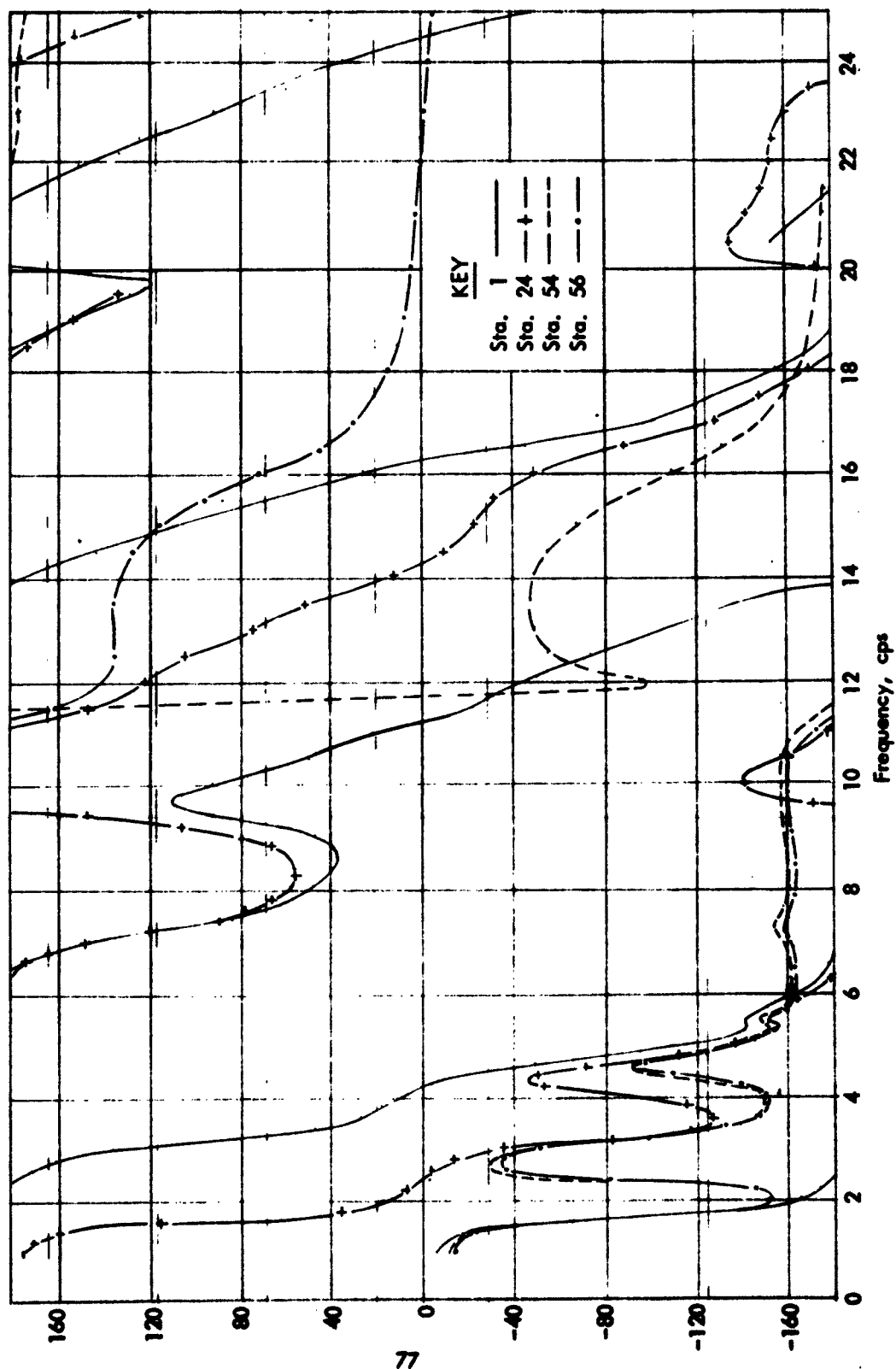


Figure 56 - Vertical Bending Phase Angle Relative to Excitation on Hull, Case 26

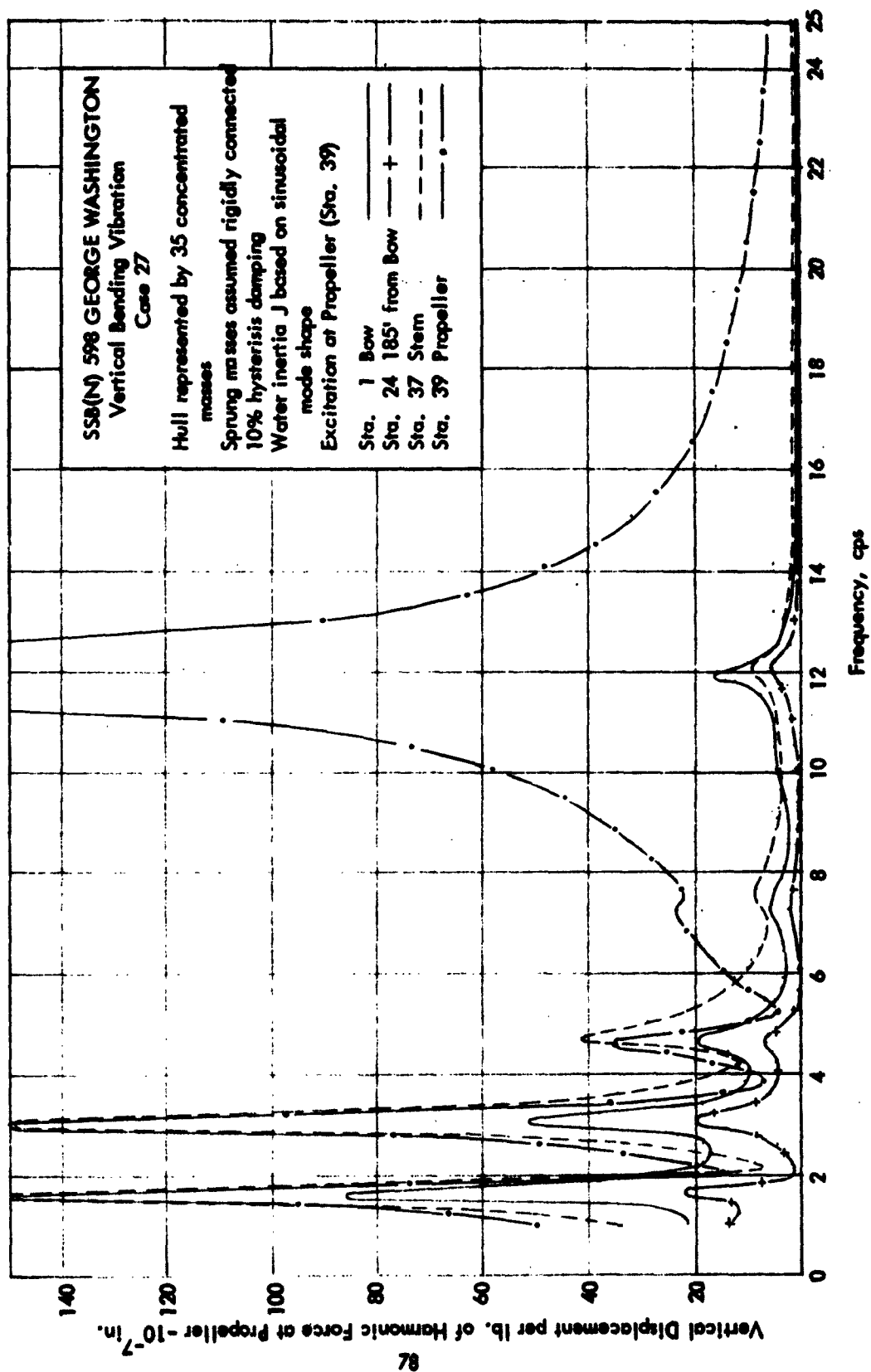


Figure 57—Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 27

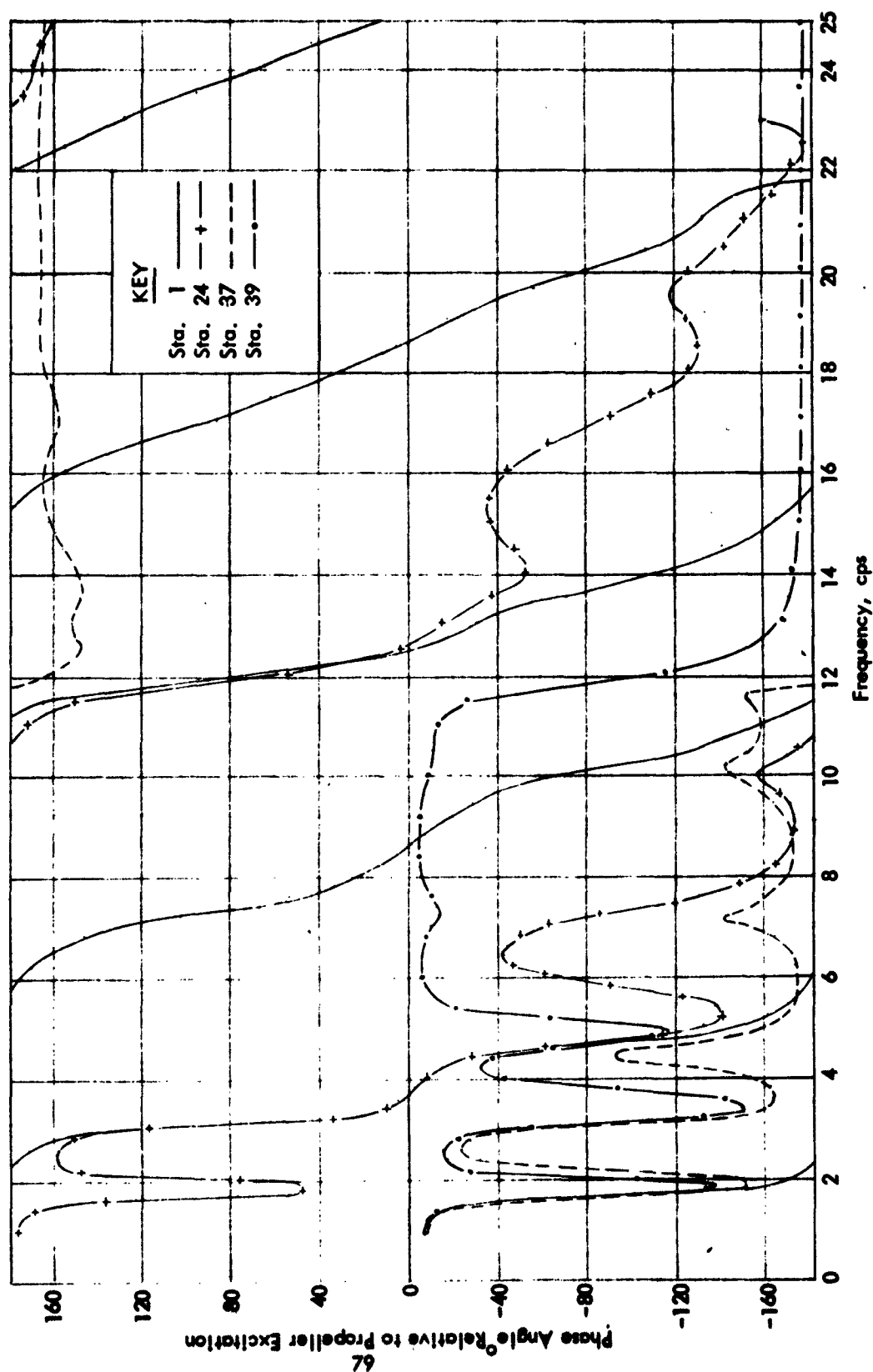


Figure 58-Vertical Bending Phase Angle Relative to Propeller Excitation, Case 27

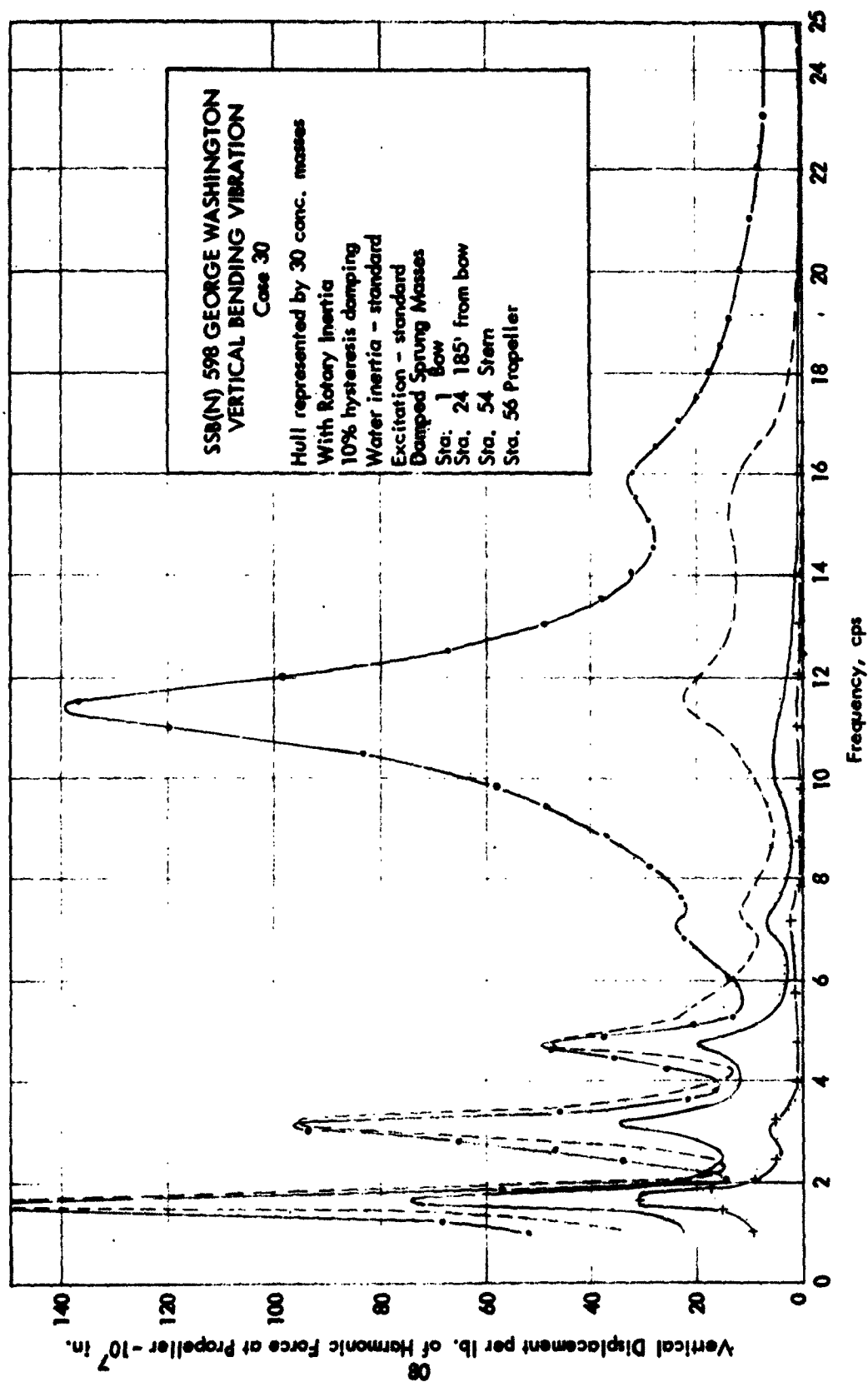


Figure 59 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 30

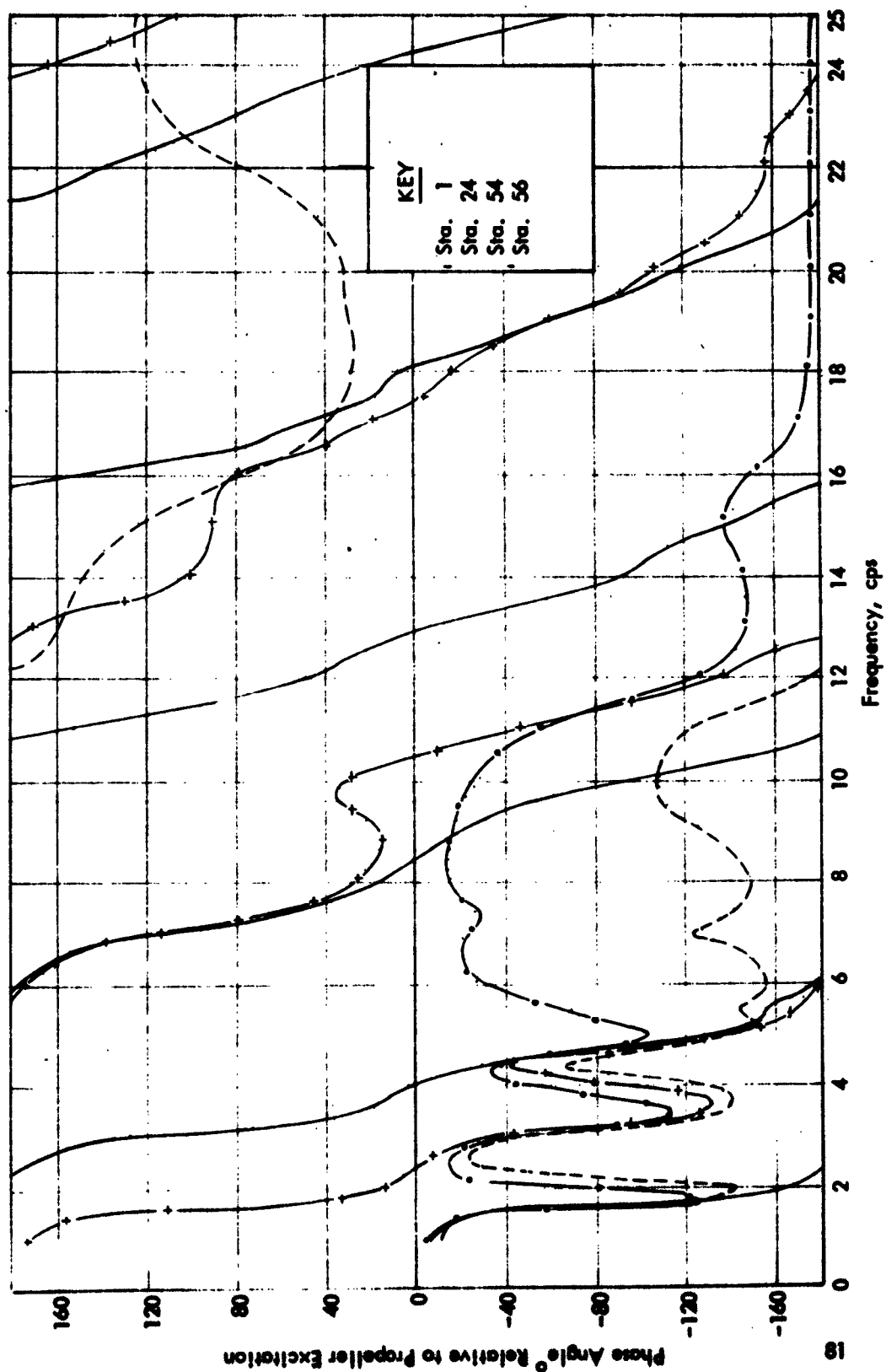


Figure 60 - Vertical Bending Phase Angle Relative to Propeller Excitation, Case 30

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sufficient to hold them within reasonable bounds even without any hull damping. Surely the effect of hull damping is two fold. (1) It reduces the amplitude of motion at the point of excitation and (2) it attenuates the relative motion of locations remote from the source of excitation. The value of damping which most closely matches that of the actual submarine will have to be obtained by comparison with the trials on the submarine. These calculations suggest that it requires less hull damping to match trial experience than was previously thought. This is a desirable result because although experience in other structures supports an assumption of 4% hysteresis-damping, it is very difficult to explain the source of 10% or more of hysteresis damping.

When comparing Cases 22, 18 and 23, representing a 10 to 1 change in the stiffness of the stern tube bearing the change in the hull response is striking. This increased stiffness bearing (3 times over the standard) moves the propeller resonance from about 11.3 cps to about 19.5 cps as might be expected. What might not be expected is the large effect of this shift upon the hull response. The amplitudes of motion at the lower resonances are very much reduced and those at the higher resonances increased by this shift. The effects of reducing the stiffness of the stern tube bearing are entirely consistent with the above results. Although the amplitude of the propeller is increased this occurs at lower speeds and there is a large decrease in the hull response in the normal operating range. These results surely emphasize the importance of treating the propeller and shafting sub-system as precisely as possible.

Case 24 indicates that the hull is less responsive to a harmonic moment (a moment arm of more than 1000 inches would be required to give equivalence) than to a harmonic force. Cases 25 and 26 as compared with Case 18 indicate that except at the propeller resonance a harmonic force at the stern will generate a slightly higher hull response than the same force at the propeller. As the location of the harmonic force is moved forward Case 26 as compared to Case 25) the response on the hull falls off noticeably.

Case 27 as compared to Case 18 shows that for the lower hull frequencies the damping associated with the sprung masses has a significant effect. It also appears that

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the inclusion of the sprung masses with their springing can change the hull frequencies by measureable amounts. However, some of the differences between Cases 27 and 18 are difficult to understand.

Finally, Case 30 as compared to Case 18, indicates that the effects of including the rotary inertia in the hull response calculations of a submarine are very small.

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## APPENDIX A

### PROCEDURE USED IN DEFINING THE SHIP WEIGHT DISTRIBUTION

The basic information used in determining the weight distribution of the ship is given in the "Detailed Weight Summary," SSB(N) 599 dated 7/1/60, prepared by the Electric Boat Division of General Dynamics Corporation.<sup>7</sup> This weight summary is in the form of an IBM print-out. The summary breaks the ship weight into about 250 classifications. Under each classification it lists the plans covering the items contributing to the weight. For each plan it gives the total weight represented by the plan and the vertical and longitudinal center of gravity.

For many of the weights, particularly heavy concentrated items, this information can be used directly for estimating the longitudinal distribution. However, for some important weight elements, such as the hull plating, the plans cover such a large portion of the ship's length that it is necessary to make independent calculations to define the weight distribution. On many weight items, such as electrical wiring, piping and joiner work, the distribution of the component weights had to be estimated from the compartments served and the longitudinal position of the center of gravity.

From the Detailed Weight Summary a chart was prepared in which the weight in each of the several classifications was distributed throughout the ship. A typical section of this chart covering the portion of the ship length between 30 feet and 80 feet from the bow is shown in Figure A-1. It consists of distributed weights and concentrated weights. The distributed weights are added up at the several sections and a curve of distributed weight prepared. The concentrated weights are listed by location and are then distributed over a two foot length. Table A-1 presents a listing of the concentrated weights over the interval covered in Figure A-1. Figure A-2 shows the distributed mass, the spread-out concentrated masses, the captured water and the water ballast for the typical section of the ship between 30 feet and 80 feet from the bow. Finally, all of these curves are added and smoothed

Account	30	40	50	60	70	80
261, 262, 263, Paint and fillers		225	239		336	
312, Stern castings, 321 Transverse Framing		2675	2540	2052	2540	3120
322, Vert. Keels and Str. 323 Bow Frm, 324 Stern Frm.						
331 Transverse Bulkhead, 342 Watertight Flats				54332 2175		
341 Inner Shell, 361 Welding	3090	3050 3300	3480	4220 6150	6850	
344 Inner Shell Inserts	5138	12158	11152	183		
351 Outer Shell, 361 Welding	3710	1930 4000	2080 2080	2240	5870 5150	8390
353 Outer Shell Inserts, 411 Foundation Mn. Propulsion			318		5980	5800

Distance From Fwd. Perp.

Figure A1 - Longitudinal Location of Hull Weights - Typical Section

Account	30	40	50	60	70	80
412 Fdn. Aux. to Mn. Prop. 413 Fdn. Misc. 414 Fdn. Elec.	73		83		227	256
415 Hull and Bkhd. Liners, 416 Shaft Tubes, 417 Fdn. Ord.	42 791	244	671	39		383
421 Pressure Proof Inclosures, 422 N.P.P. Inclosures	3358	1291	1511	486	3520	1240
423 Secondary Hull Div., 424, Shield Supp. and Inclosure		162	426			
431 Superstructure Framing						
432 Superstructure Plating						
433 Fdn. Ext. to Hull, 434 S. S. openings, 436 S. S. Covering		223			144	
443 Guards and Fairwater, 444 Stabl.						

Distance From Fwd. Perp.

Figure A-1 (continued)

Account	30	40	50	60	70	80
452, 453, 455 Platform Decks and Covering	128	354	622	396		1181
462, 463, 465 Fairwater Bridge, Trackers, Fdn.						
511, Rudders, 513 Stem Diving Pl., 515 Fairwater Planes						
521 Gages, 524 Pipe Hangers, 531 Mast and Spars			25			
532 Liferails, 534 P. P. Lockers, 535 Hull Protectors			13			
537 Ladders and Stairs, 538 Label Plates, 539 Fittings		6	159			
541 P. P. Hatch Trunks, 542 P. P. Doors			768			
543 P. P. Man holes, 544 Ld. Hatches, 545 Non P. P. openings	182		1135	135		25

Distance From Fwd. Perp.

Figure A-1 (continued)

Account	30	40	50	60	70	80
547 P.P. Hatch Covers, 551 Flood Valves, 101 552 Ballast Tank Vents		213		197		
553, 554 Sea Connections, Ship and Machinery	141		5		14	
561 Lead Ballast	4220 1193	37200	58,650			2210 12940
571, 572, 573, 574 Joiner work, Lockers, Berthing	64		408		880	
575, 576 Insulation, Thermal and Acoustic	338		520		665	
577 Galley and Laund. Appl. 578 Health Physics					328	
611, Windlass, 613 Anchors, 614 Capstons, 617, 619 Gear	1348	2474	1348			
621, 622, 624, 625, 625 Steering and Diving Gears						

Distance From Fwd. Perp.

Figure A-1 (continued)

[illegible]

Distance From Fwd. Perp.

**Figure A-1 (continued)**

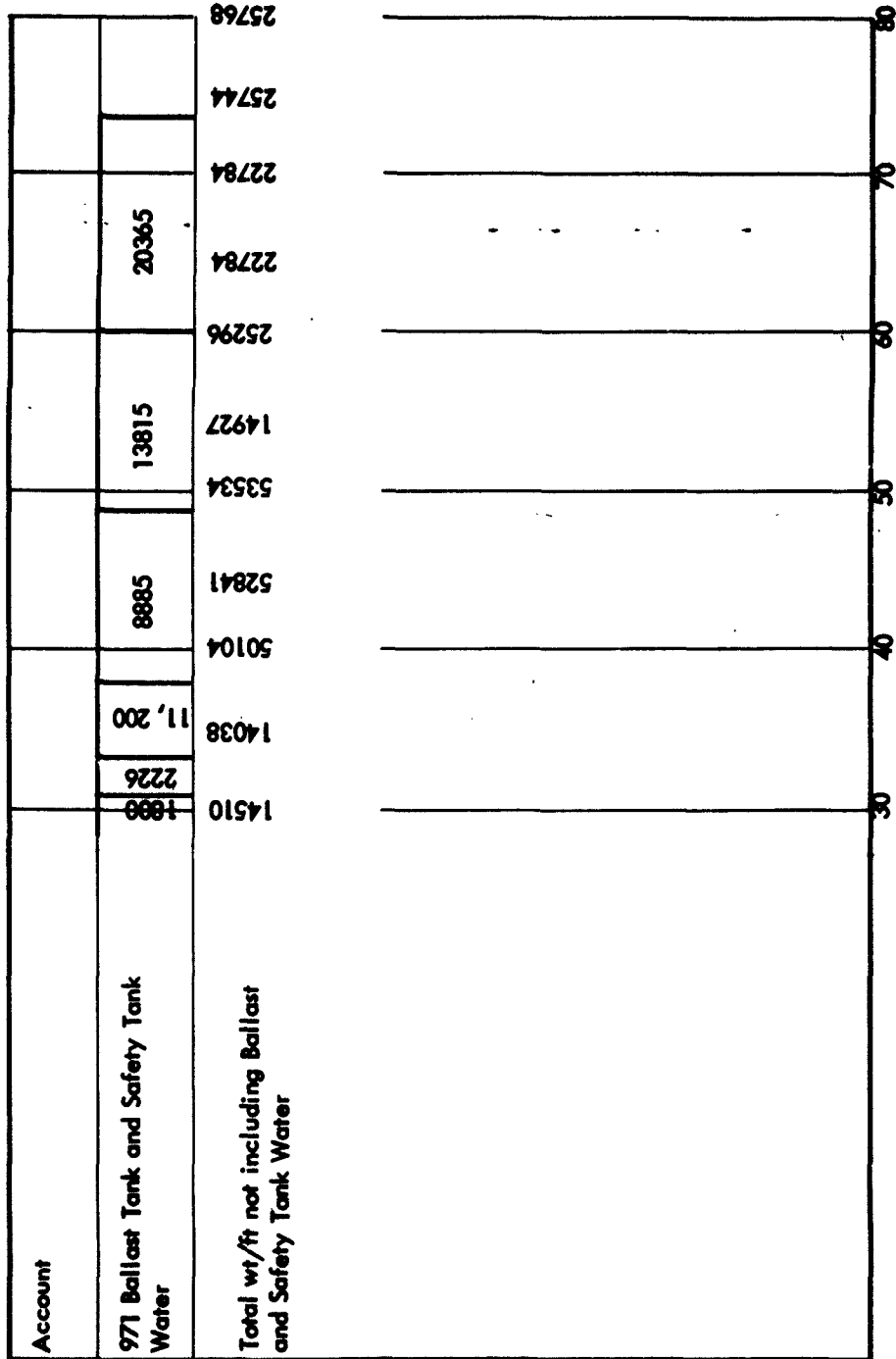
Account	30	40	50	60	70	80
718 732 Mn. Prop. Cond., Stuff. Bxs., Cooling Control, Adjuncts., L. O. Prop. Motor and Contr.					35	
733 Switch Bds., 734 Pwr. Wiring, 737 Wireways, 738 Sw. Bds.	41	107	176	262		
741, 742, 743, 744, 746, 747, 748, 749 Mn. and Aux Steam, Cond., Feed Drain, Insul. and Control						
751 → 758 Batteries and Accessories						7550
761 → 767 H. P. and L. P. air, compr. piping	168	794	865	155	168	71
771 → 779 Aux. Pwr. Sys.						
781 → 789 Reactor and Primary Plant						
791 → 793 Hydraulic Pwr. Plants	14	22	12			

**Figure A-1 (continued)**

Account	30	40	50	60	70	80
821→851 Torpedo Tubes and Assoc. Equipment	2312	1707	1857	986		
853→868 Missile Tubes and Fire Control (863 carried with 962)						
901→908 Repair Parts	230		421		780	
920→965 Small Arms, Stores, Provisions	3		1103		853	
966→968 + 863 Ammun., Missiles, Torpedoes, Ord. Stores	1142		1217	75	58	
972→985 Water, Air, Oil, etc.		125	315	524	399	51
987 Officers, crew and effects		38	63		259	1768
991→999 Fixed Liquids	725	819	725	160	162	68
				2451		

Distance From Fw. Perp.

Figure A-1 (continued)



Distance From Fwd. Perp.

Figure A-1 (continued)

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**TABLE A-1 - SSB(N) 598 - WEIGHTS LOCATED BETWEEN 30' and 80' FROM THE BOW THAT ARE TREATED AS CONCENTRATED WEIGHTS**

Distance from Bow ft.	Weight Account Number	Weight lbs	Remarks
30	682→696	1708	Fws. Escape Trunk
30.5	541→542	9015	
33	344	5139	
37	344	2158	
36	611→619	1348	
36.5	421	3358	
41.6	417	791	
42	344	1152	
42	353	318	
42	421	1297	
42	611→619	2474	
42.5	433	223	
44.8	417	244	
50	421	25	
50.5	611→619	1348	
53.4	537	159	Oxy. Tanks and Piping
53.4	682→696	205	
54	543→545	155	
54	641→644	520	
54	648→649	11,500	
56	344	83	
56	422	151	
56	627→634	522	
56	451→462	2937	
56.5	541→542	768	
56.5	543→544	155	

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TABLE A-1 - (continued)

Distance from Bow ft.	Weight Account Number	Weight lbs	Remarks
57.5	415	671	1007
57.5	733→738	176	
57.5	761→767	160	
58.6	331	54,332	Trans. Bkhd.
59	991→999	2457	Lead Ballast
61	641→644	1984	
74	561	12,940	
74.5	353	1383	
75	421	936	

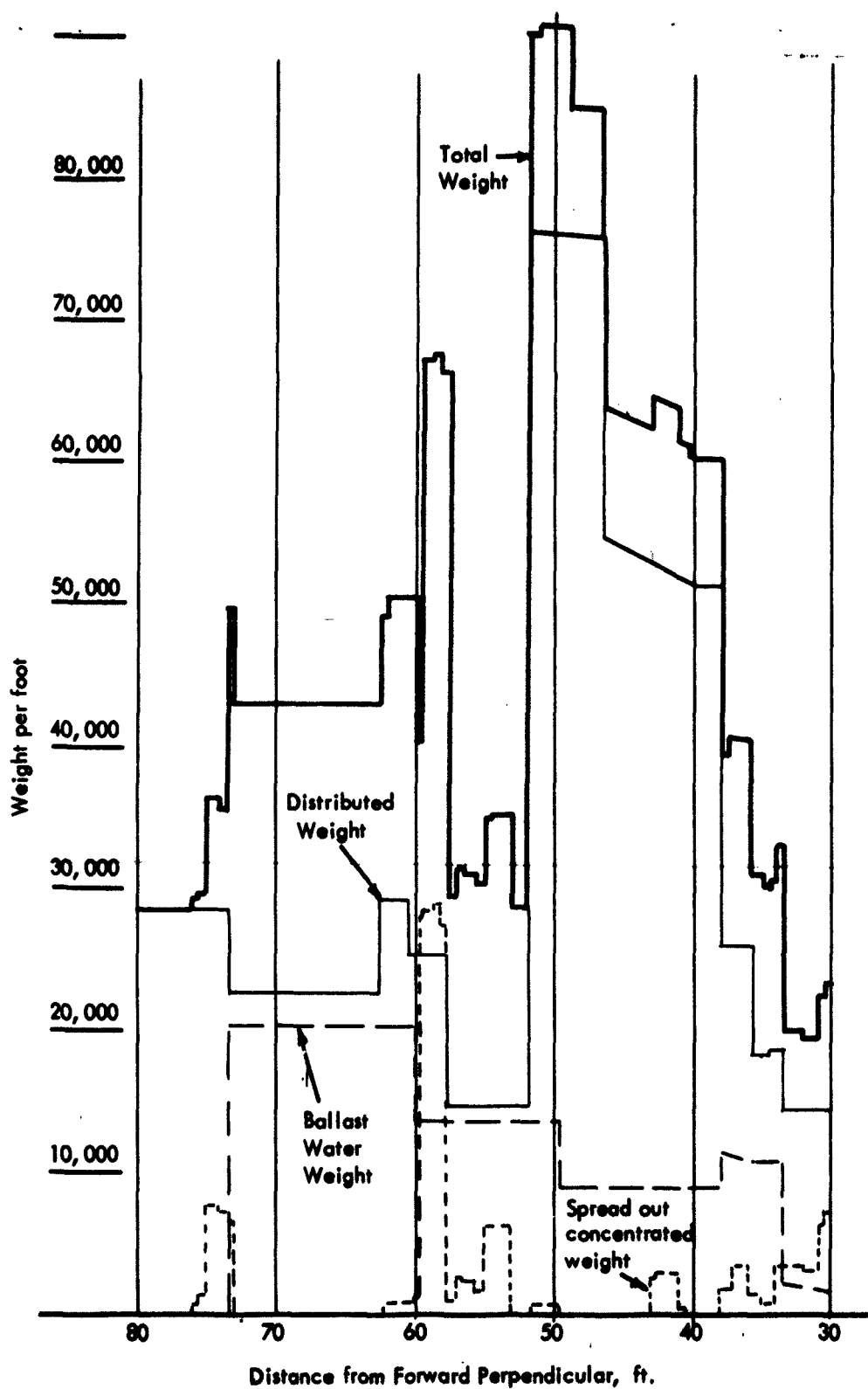


Figure A-2 - Section of Weight Distribution Curve  
A-12

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slightly and the resulting curve used to represent the weight distribution on the ship. This curve is given in Figure 3. For a check this curve is integrated and, with captive water deducted, compared with the submerged displacement of the ship.

With the general process explained, we will now discuss some of the details of the process of distributing the weights along the hull. For some of the main categories of hull weight it is necessary to compute the weight distribution along the hull. This is not particularly laborious in the case of the inner and outer hull shell plating, since the areas must be computed in the stiffness determination. Lead ballast is a very significant item of weight and can be concentrated within a small length. It is therefore well to define this weight closely from the plans. Also the water ballast is a very significant weight item. Normally, this is easily defined from the weight in a particular tank but in the case of the **GEORGE WASHINGTON** it was necessary to study the distribution of weight in the forward ballast tanks. Items like watertight flats are usually broken down into separate compartments and can be distributed quite easily by brief reference to the plans to ascertain the location, extent and shape. The framing is broken down in the Detailed Weight Summary into groups of four or five frames and the weights of these are quite easily distributed over their longitudinal span.

The distribution of the weights under some classifications such as furniture, wiring, and piping are more difficult to make. Where it is clear from the plan title that the weights represented by the plan fall within a certain span of length, they can be distributed either uniformly or by some simple distribution over the length of the span. For convenience in determining weight distribution for such weight categories as interior painting, electrical wiring, telephone wiring, etc., the interior of the hull was considered to be divided in ten sections as indicated in Table A-2. Using these sections it is possible to estimate a weight factor that will indicate what proportion of the weight will fall within each section. The application of this process to the interior paint, account 261, which weighs 28,207 pounds is illustrated in Table A-3. The application of the process to the distribution of the weight of power wiring through the ship (weight about 45,000 lbs) is illustrated in Table A-4. The

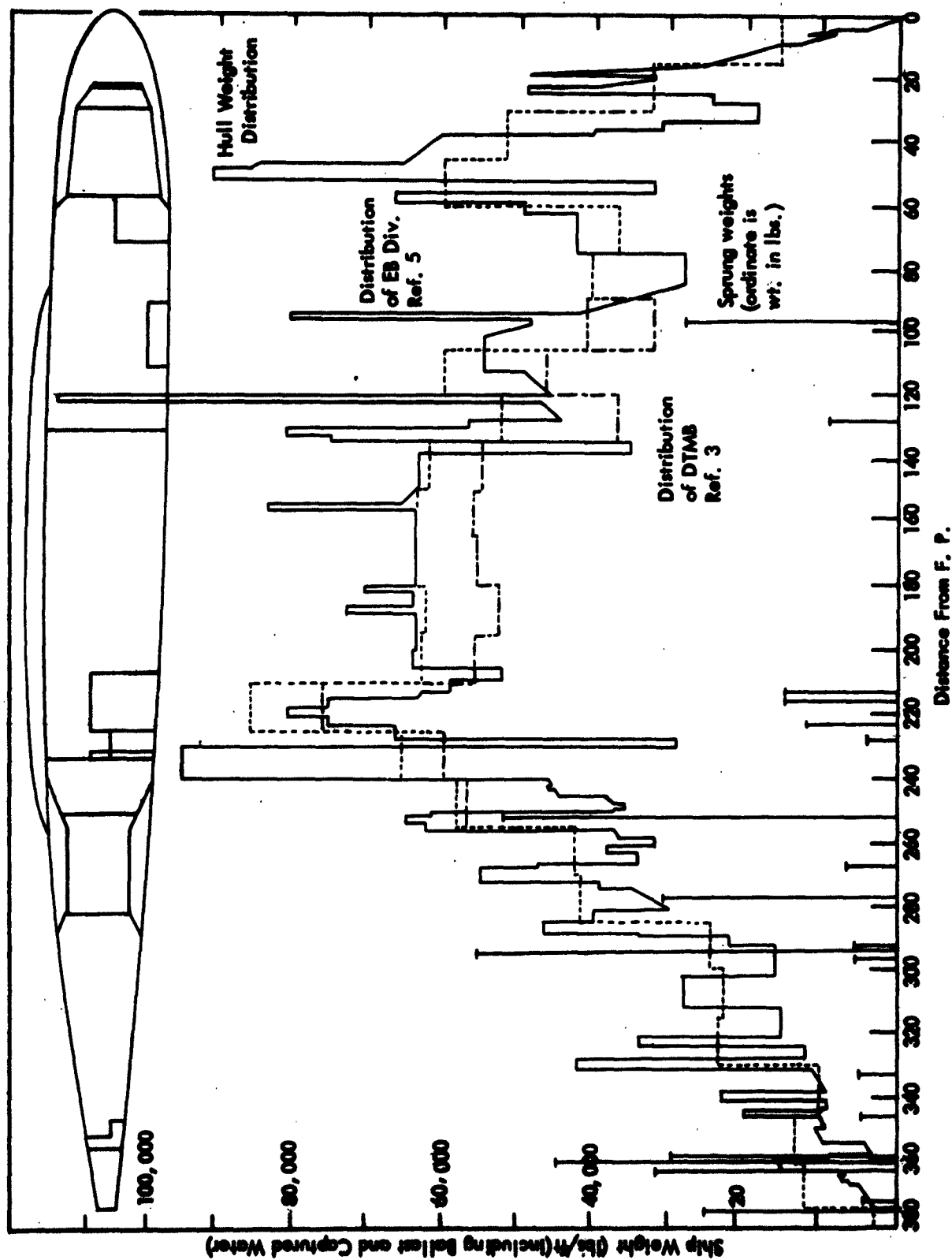


Figure 3 - Submerged Hull Weight SSN(N) 598 GEORGE WASHINGTON

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**TABLE A-2 - HULL DIVISIONS FOR ESTIMATING WEIGHT DISTRIBUTION**

<u>Section</u>	<u>Compartment</u>	<u>Fr. Nos.</u>	<u>Distances from F. P.</u>	<u>Length</u>
0	Bow	0 → 13	0 → 25.25	25.25
1	Torpedo Rm. Fwd.	→ 20	→ 40.83	15.58
2	Torpedo Rm. Aft.	→ 26	→ 57.50	16.67
3	Living Qtrs.	→ M3	→ 92.75	35.25
4	C. O. C.	→ M17	→ 132.92	40.17
5	Missile Rm.	→ M44	→ 208.92	76.0
6	Missile Compt. Aft.	→ 44	→ 233.67	24.75
7	Reactor Compt.	→ 52	→ 253.67	20.00
8	Aux. Mach. Space	→ 64	→ 285.67	32.0
9	Engine Rm. Fwd.	→ 77	→ 320.17	34.50
10	Engine Rm. Aft.	→ 92	→ 353.67	33.50
11	Stern	→ 102	→ 373.50	19.83

**TABLE A-3 - DISTRIBUTION OF WEIGHT OF INTERIOR POINT**

<u>Compartment</u>	<u>Length</u>	<u>Weighting Factor</u>	<u>Product</u>	<u>wt/ft</u>
1	15.6	0.7	10.9	34
2	16.7	1	16.7	48
3	35	3	105	145
4	40	3	120	145
5	76	2	152	97
6	25	1	25	48
7	20	1	20	48
8	32	1.5	48	73
9	34	1.5	51	73
10	34	1	34	48
			<u>583</u>	

$$\text{wt/ft} = \frac{28,207}{583} \times \frac{\text{product}}{\text{length}} = 48.4 \text{ weighting factor}$$

**TABLE A-4 - DISTRIBUTION OF WEIGHT OF POWER WIRING**

<b>Assume:</b> a 1 cable from compartment 2 to compartment 9 b 1 cable from compartment 3 to compartment 9 c 2 cables from compartment 4 to compartment 9 d 1 cable from compartment 5 to compartment 9 e 2 cables from compartment 6 to compartment 9 f 1 cable from compartment 7 to compartment 9 g 2 cables from compartment 8 to compartment 9 h 3 cables in compartment 9 i 3 cables in compartment 10 to compartment 9											
wt/ft	compartment	a	b	c	d	e	f	g	h	i	Total
25	2	17									17
50	3	35	35								70
99	4	40	40	80							160
124	5	76	76	152	76						380
148	6	25	25	50	25	50					175
198	7	20	20	40	20	40	20				160
248	8	32	32	64	32	64	32	64			320
320	9	34	34	68	34	68	34	68	102	100	440
73	10									100	100
											<u>1822</u>

$$\text{wt/ft in compartment} = \frac{\text{Total}}{\text{Grand Total}} \times \frac{45,000}{\text{Length of compartment}}$$

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application to other categories such as the announcing system, the telephone system, etc., can be understood.

Equipment weights which exceed 2000 pounds and are shock mounted are treated as sprung masses (See Appendix G). The fairwater planes, the stabilizers and the stern planes are especially studied as elastic structures in vertical vibration but are considered as rigid masses in the axial direction. The rudders are considered to be rigidly connected in both vertical and longitudinal vibration. Properly, the stabilizers and control surfaces should have been incorporated in the calculation program as elastic sub-systems but since the program was not developed when the hull was defined, this was not done.

The "captured" water, water in the free flooding sections of the ship, is treated as part of the mass of the ship. Consider, for example, the water between the superstructure and the hull. Beneath the superstructure the water has a relatively free flowing space. However, the air vent in the superstructure deck is only a very small percentage (less than 5%) of the superstructure deck area. Since it normally passes air this area is adequate for venting but water flowing through it would have to pass at more than 20 times the mean velocity in the tank. As a consequence for any vibration when the submarine is under water the water cannot flow through the superstructure vents in sufficient quantity to allow any significant free flow and the water is indeed "captured" and forced to move with the ship.

## APPENDIX B

### VIRTUAL INERTIA OF ENTRAINED WATER IN LONGITUDINAL MODES

#### 1. Determination of an Ellipsoid to Represent the Hull

The distribution of the inertia of the entrained water on the SSB(N) 598 is assumed to be the same as that of the ellipsoid of revolution which most closely fits the hull. The first step in determining the inertia of the entrained water therefore is to find the dimensions of the ellipsoid which most closely matches the hull. The process that is used is the "Method of Least Squares" and is described in Reference 8.

Although the axes of the portions of the submarine that are surfaces of revolution are offset in the vertical direction and some of the cross-sections are not circular, it was assumed that the prolate ellipsoid which matches the breadth of the submarine most closely would adequately represent the submarine. The origin of the axes of coordinates was taken at the centerline at the bow and the x direction was taken along the centerline of the bow and stern sections. The equation of the ellipse that rotates into the ellipsoid is  $\frac{(x-b)^2}{a^2} + \frac{y^2}{c^2} = 1$  where a, b and c are to be determined by the method of least squares to give the ellipsoid that most closely approximates the submarine.

Assume for a first approximation that  $a = b = 190'$  and  $c = 16.5'$ .

From the equation of the ellipse

$$y = c \left( 1 - \frac{(x-b)^2}{a^2} \right)^{1/2}$$

Expand this value of y in a Taylor's series.

$$y = y_{a,b,c} + \frac{\partial y}{\partial a}_{a,b,c} \Delta a + \frac{\partial y}{\partial b}_{a,b,c} \Delta b + \frac{\partial y}{\partial c}_{a,b,c} \Delta c + \dots$$

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where:

$$\frac{\partial y}{\partial a} = c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)^2}{a^3}$$

$$\frac{\partial y}{\partial b} = c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)}{a^2}$$

$$\frac{\partial y}{\partial c} = \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{1/2}$$

Letting  $s$  equal the sum of the squares of the differences,  $v_i$ , between the actual value of  $y$ ,  $y_i$ , and the value computed by the equation for  $y$ .

$$s = \sum_{i=1}^n v_i^2 = \sum_{i=1}^n (y - \tilde{y}_i)^2$$

$$= \sum_{i=1}^n \left\{ c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{1/2} + c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)^2}{a^3} \Delta a + c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)}{a^2} \Delta b + \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{1/2} \Delta c - \tilde{y}_i \right\}^2$$

The values of  $\Delta a$ ,  $\Delta b$  and  $\Delta c$  which will minimize  $s$  are found by setting

$$\frac{\partial s}{\partial (\Delta a)} = \frac{\partial s}{\partial (\Delta b)} = \frac{\partial s}{\partial (\Delta c)} = 0$$

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This yields the normal equations:

$$\sum_{i=1}^n v_i c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)^2}{a^3} = 0 \quad B(1)$$

$$\sum_{i=1}^n v_i c \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)}{a^2} = 0 \quad B(2)$$

$$\sum_{i=1}^n v_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{1/2} = 0 \quad B(3)$$

After substituting the value of  $v_i$ , these equations reduce to:

$$\begin{aligned} & c \sum_{i=1}^n (x-b)^2 + \frac{c}{a^3} \Delta a \sum_{i=1}^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^4 \\ & + \frac{c}{a^2} \Delta b \sum_{i=1}^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^3 + \Delta c \sum_{i=1}^n (x-b)^2 \\ & - \sum_{i=1}^n y_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} (x-b)^2 = 0 \end{aligned} \quad B(1a)$$

$$\begin{aligned} & c \sum_{i=1}^n (x-b) + \frac{c}{a^3} \Delta a \sum_{i=1}^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^3 \\ & + \frac{c}{a^2} \Delta b \sum_{i=1}^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^2 + \Delta c \sum_{i=1}^n (x-b) \end{aligned}$$

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$$-\sum_i^n y_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} (x-b) = 0 \quad B(2a)$$

$$\begin{aligned} & c \sum^n \left[ 1 - \frac{(x-b)^2}{a^2} \right] + \frac{c}{a^3} \Delta a \sum^n (x-b)^2 \\ & + \frac{c}{a^2} \Delta b \sum^n (x-b) + \Delta c \sum^n \left[ 1 - \frac{(x-b)^2}{a^2} \right] \\ & - \sum_i^n y_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{1/2} = 0 \quad B(3a) \end{aligned}$$

To find the values of  $\Delta a$ ,  $\Delta b$  and  $\Delta c$ , tables are formed for the quantities

$$\begin{aligned} & \sum^n (x-b), \sum^n (x-b)^2 \\ & \sum^n \left[ 1 - \frac{(x-b)^2}{a^2} \right] \\ & \sum^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^2, \sum^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^3, \sum^n \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1} (x-b)^4 \\ & \sum_i^n y_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2}, \sum_i^n y_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} (x-b) \\ & \sum_i^n y_i \left[ 1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} (x-b)^2 \end{aligned}$$

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Values of  $y_i$  are picked from the graph of the half breadths at the 15' - 9-7/8" water line as plotted in Figure B-1 for six values of  $x$ .

The results are given in Table B-1.

Substituting the values from Table B-1 into the equations B(1a), B(2a) and B(3a) yields the following:

$$17,380 \Delta a - 5690 \Delta b + 88,700 \Delta c = 146,000 \quad \text{B(1b)}$$

$$0.2138 \Delta a - 0.00456 \Delta b + 3.543 \Delta c = 2.0 \quad \text{B(2b)}$$

$$29.95 \Delta a - 132 \Delta b + 10 \Delta c = 2152 \quad \text{B(3b)}$$

From a simultaneous solution of these:

$$\Delta a = 0.517' \quad a \text{ improved} = 190' + 0.517'' = 190.52'$$

$$\Delta b = -16.12' \quad b \text{ improved} = 190' - 16.12' = 173.88'$$

$$\Delta c = 0.515' \quad c \text{ improved} = 16.50' + 0.515' = 17.015'$$

Therefore the ellipsoid that most closely matches the submarine is:

$$\frac{(x-173.88)^2}{(190.52)^2} + \frac{(y^2 + z^2)}{(17.015)^2} = 1$$

The curve of this quantity is plotted in Figures B-1 and B-2. It is obvious that although this ellipsoid matches the ships hull as closely as is possible by any single ellipsoid, there is still a considerable discrepancy at the ends.

Because of the complexity of dealing with a closer simulation of the ship, the simple ellipsoid has been considered to be an adequate representation at the present time. The methods of Reference 9 would have to be developed further before they would be applicable to the problem of a shape differing from an ellipsoid.

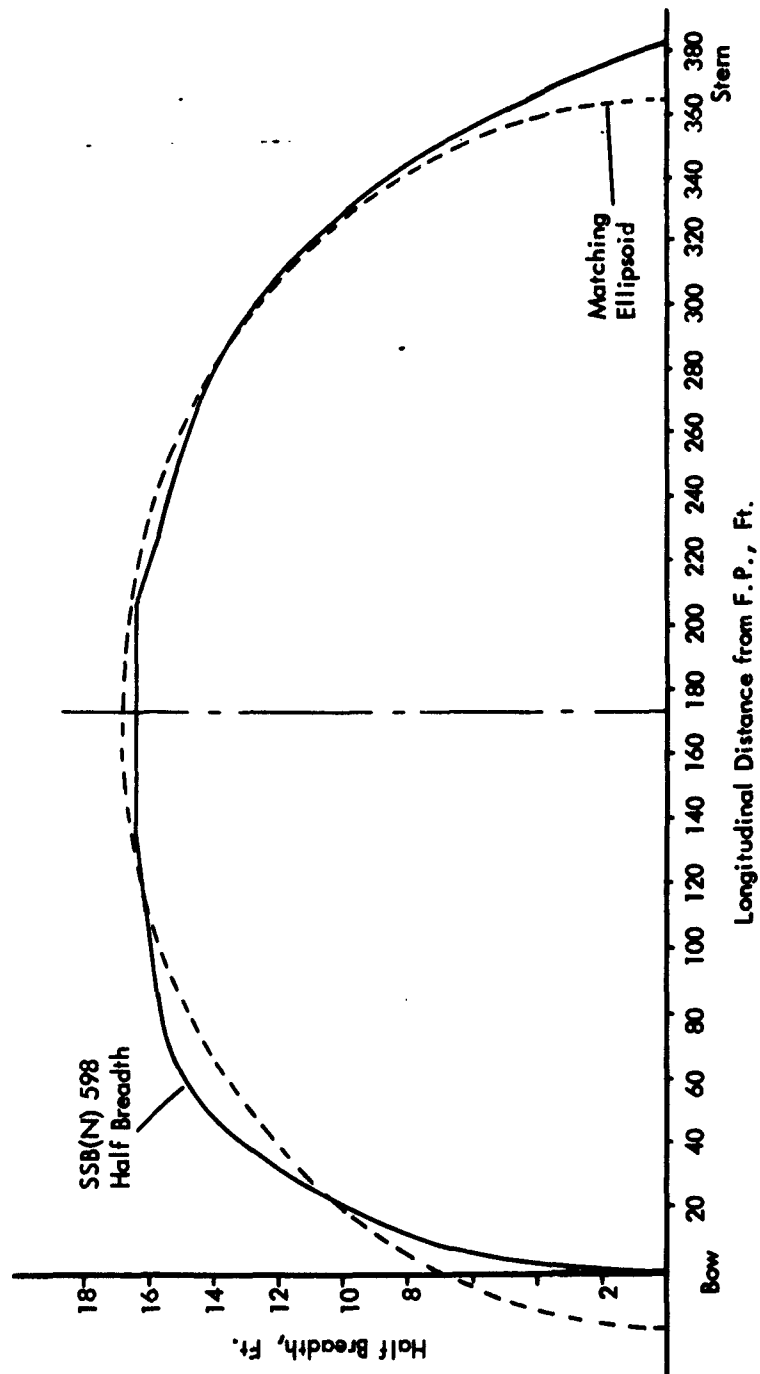


Figure B-1 - Half Breadth of SSB(N) 598 at 15' - 9-7/8" Water Line and Radius of Matching Ellipsoid

TABLE B-1 - MEAN SQUARE DETERMINATION OF ELLIPSOID

x	$\gamma_1$	(x-b)	$(x-b)^2$	$\left[1 - \frac{(x-b)^2}{a^2}\right]$	$\left[\frac{-1}{\gamma_1} (x-b)^2\right]$	$\left[\frac{-1}{\gamma_1} (x-b)^3\right]$	$\left[\frac{-1}{\gamma_1} (x-b)^4\right]$
20	10.0	-170	28,900	0.20	144,500	-24.6x10 <sup>6</sup>	4,180x10 <sup>6</sup>
60	15.0	-130	16,900	0.531	31,780	-4.13x10 <sup>6</sup>	537x10 <sup>6</sup>
150	16.5	-40	1,600	0.956	1,670	-0.066x10 <sup>6</sup>	2.7x10 <sup>6</sup>
250	15.4	+60	3,600	0.902	3,990	+244x10 <sup>6</sup>	14.4x10 <sup>6</sup>
300	12.9	+110	12,100	0.664	18,200	2.00x10 <sup>6</sup>	220x10 <sup>6</sup>
350	7.1	+160	25,600	0.290	88,400	14.12x10 <sup>6</sup>	2,260x10 <sup>6</sup>
		-10	88,700	3.543	288,540	-12.43x10 <sup>6</sup>	7.214x10 <sup>6</sup>

x	$\gamma_1$	$\left[1 - \frac{(x-b)^2}{a^2}\right]^{1/2}$	$\left[\frac{1}{\gamma_1}\right]$	$\left[\frac{-1}{\gamma_1} (x-b)\right]$	$\left[\frac{-1}{\gamma_1} (x-b)^2\right]$
20	10.0	.447	4.47	-3800	646,000
60	15.0	.730	10.93	-2670	347,000
150	16.5	.978	16.15	-675	27,000
250	15.4	.950	14.62	+973	58,300
300	12.9	.815	10.51	1,740	191,300
350	7.1	.538	3.82	2,115	338,000
		60.50	-2,317	1,607,600	

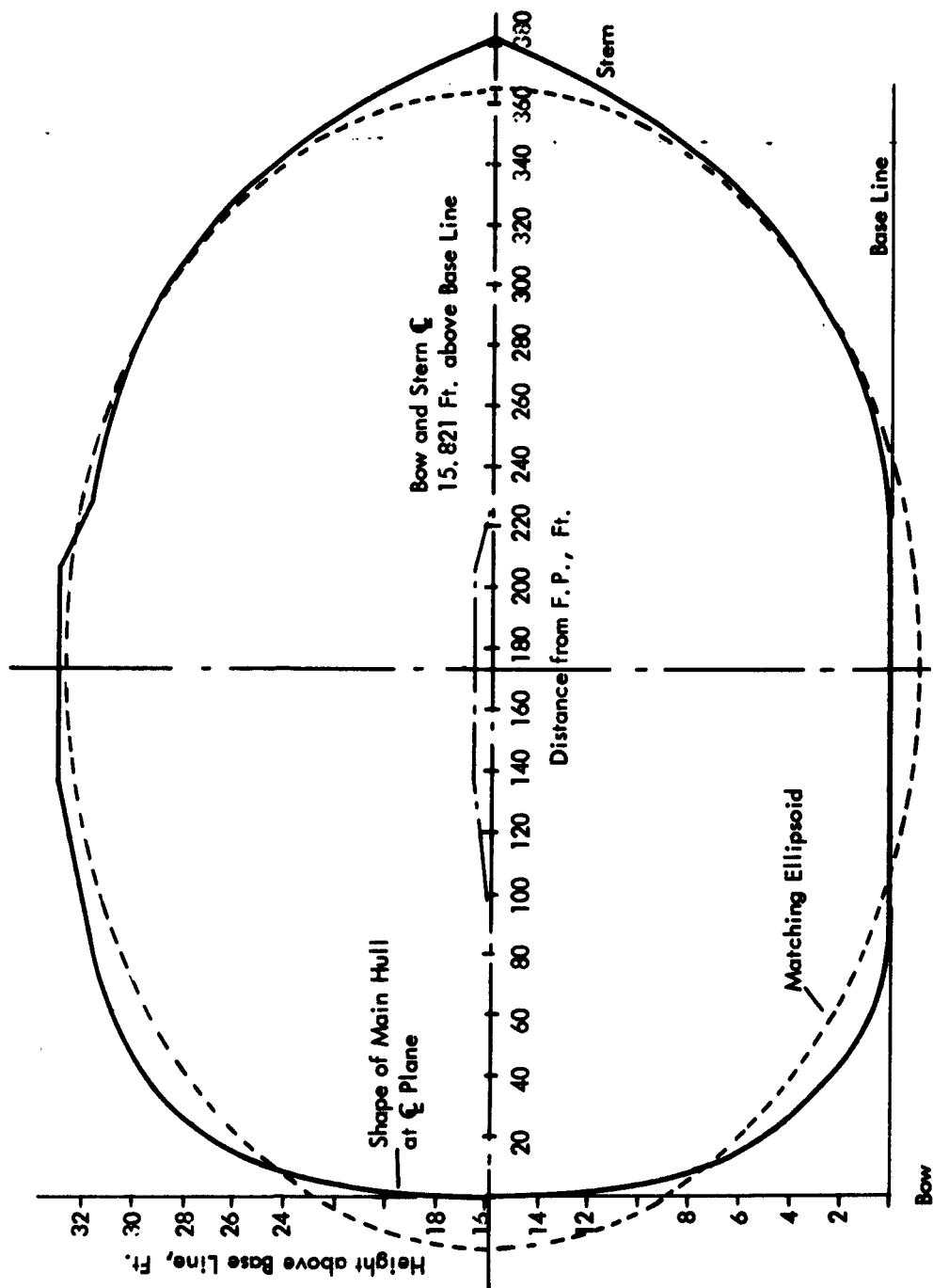


Figure B-2 - Match of Ellipsoid with Vertical Projection of Main Hull

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2. Analytical Representation of the Vibration Pattern

It is also necessary in the considerations of equivalent mass to work with a pattern of vibration. For this purpose the nodal pattern at the natural frequency of 10.40 cps, as computed for the mass and stiffness reported in Reference 3 was used. This nodal pattern is plotted in Figure B-3. For convenience in the computations a deflection curve of the form  $\delta = a_1 (173.88 - x) + a_3 (173.88 - x)^3$

$$= a_1 x' + a_3 x'^3$$

was assumed and the values of  $a_1$  and  $a_3$  to match the experimental curve determined by the method of Least Squares. The resulting analytical expression for the deflection is:

$$\delta = 4.11 \times 10^{-6} - 11.08 \times 10^{-12} (x')^3$$

This is plotted on Figure B-3 to indicate the adequacy of the match.

3. The Distribution of the Entrained Water Inertia on the Ellipsoid

We are now concerned with the water inertia of an ellipsoid of

major axis,  $a_1 = 190.52'$

minor axes,  $b_1$  and  $c_1 = 17.015'$

vibrating in a longitudinal mode such that  $\delta = 4.11 \times 10^{-6} x - 11.08 \times 10^{-12} x^3$

where  $x$  is measured from the center of the ellipsoid.

The velocity in the longitudinal direction is  $\approx j\omega\delta$

Express this as:

$$u_e = Ax - Bx^3$$

Because of the poisson ratio effects there will be a radial velocity to the hull associated with the longitudinal velocity.

$$\text{The longitudinal strain} = \frac{\partial \delta}{\partial x}$$

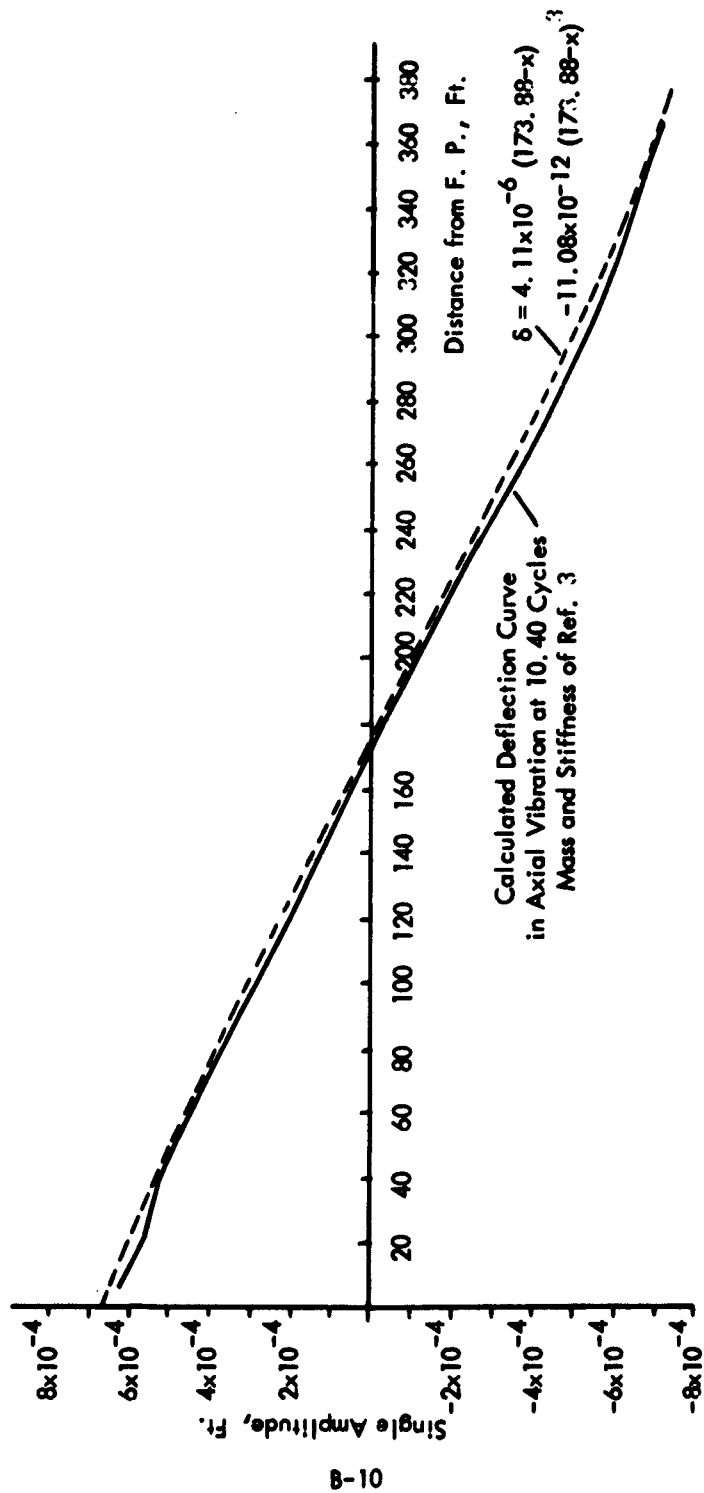


Figure B-3 - Representation of Normal Mode Deflection by a Functional Relationship

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Corresponding to this longitudinal strain there will be a poisson strain  $\sigma \frac{\partial \delta}{\partial x}$ , where  $\sigma$  is a ratio that depends upon the poisson ratio of the material of the submarine and the ratio of frame area to shell area. Hence the poisson velocity =  $y$  (or  $z$ )  $\sigma \frac{\partial \delta}{\partial x}$  and

$$v_e = \sigma y (A - 3 Bx^2)$$

$$w_e = \sigma z (A - 3 Bx^2)$$

The methods used for determining the water inertia were originally developed by Lamb<sup>10</sup> and are well presented in Reference 11. The methods involve the expression of the properties of the ellipsoid and the vibration in terms of ellipsoidal coordinates. In this manner a potential function can be obtained. In most works with virtual inertia it is possible through energy considerations to obtain an equivalent mass for the water vibrating with the ellipsoid. However, in this case the effects of the entrained water are distributed over the full length of the ellipsoid and in order to find the effects of this entrained water at any section of the hull it is necessary to compute at each longitudinal location the component of the force associated with the dynamic pressures between the hull and the water that result from the modal pattern. This force, when related to the amplitude of vibration at this location, can be used to determine the equivalent mass of the water at this location.

The discussion of the geometry and the kinematics of a vibrating ellipsoid given in Reference 11 is directly applicable to this problem and therefore will not be repeated. Because the concern of this study is axial vibration rather than bending vibrations the applications of the general theory differ.

Express the velocities in ellipsoidal coordinates

$$\begin{aligned} u_e &= Ax - Bx^3 \\ &= Ak_u \zeta_o - B (k_u \zeta_o)^3 \end{aligned}$$

$$\begin{aligned}
 v_{\theta} &= \sigma y (A - 3 B x^2) \\
 &= \sigma k (1 - \mu^2)^{1/2} (\xi_0^2 - 1)^{1/2} \cos \theta \left[ A - 3 B (k \mu \xi_0)^2 \right] \\
 w_{\theta} &= \sigma k (1 - \mu^2)^{1/2} (\xi_0^2 - 1)^{1/2} \sin \theta \left[ A - 3 B (k \mu \xi_0)^2 \right]
 \end{aligned}$$

Since these velocities are those in the surface of the ellipsoid this indicated by  $\xi_0$ , the ellipsoid representing the hull.

$$\begin{aligned}
 F(\mu, \theta) &= \mu v_{\theta} + (1 - \mu^2)^{1/2} \xi_0 (\xi_0^2 - 1)^{-1/2} (v_{\theta} \cos \theta + w_{\theta} \sin \theta) \\
 &= \mu \left[ A k \mu \xi_0 - B (k \mu \xi_0)^3 \right] \\
 &\quad + (1 - \mu^2)^{1/2} \xi_0 \sigma k (1 - \mu^2)^{1/2} \left[ A - 3 B (k \mu \xi_0)^2 \right] \\
 &= \mu \left[ A k \mu \xi_0 - B (k \mu \xi_0)^3 \right] + \sigma k \xi_0 (1 - \mu^2) \left[ A - 3 B (k \mu \xi_0)^2 \right] \\
 &= A \left[ k \mu^2 \xi_0 + \sigma k \xi_0 (1 - \mu^2) \right] - B \left[ (k \mu \xi_0)^3 + 3 \sigma k \xi_0 (1 - \mu^2) k \mu \xi_0^2 \right] \\
 &= A \left[ k \mu^2 \xi_0 (1 - \sigma) + \sigma k \xi_0 \right] - B \left[ (k \mu \xi_0)^3 \mu (1 - 3\sigma) + \frac{3}{\mu} \sigma (k \mu \xi_0)^3 \right]
 \end{aligned}$$

Applying the condition that  $k F(\mu, \theta) = \frac{\partial \phi}{\partial \xi}$ , to the general solution for velocity potential in ellipsoidal coordinates, i. e.,

$$\phi = \sum_{n=0}^{\infty} \sum_{s=0}^{\infty} (B_n^s \sin s\theta + C_n^s \cos s\theta) P_n^s(\mu) Q_n^s(\xi)$$

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leads to

$$\sum_{n=0}^{\infty} \sum_{s=0}^n (B_n^s \sin s\theta + C_n^s \cos s\theta) P_n^s(\mu) \dot{Q}_n^s(\xi_0) = kF(\mu, \theta)$$

Since the  $F(\mu, \theta)$  is independent of  $\theta$  this condition is only met where  $s=0$

Therefore  $B_n^s$  falls out of the solution

$$C_n^s \text{ becomes } C_n$$

$$P_n^s \text{ becomes } P_n$$

$$\dot{Q}_n^s \text{ becomes } \dot{Q}_n \text{ and the equation becomes,}$$

$$\sum_{n=0}^{\infty} C_n P_n(\mu) \dot{Q}_n(\xi_0) = kF(\mu, \theta)$$

Hence

$$C_n = \frac{2n+1}{4\pi \dot{Q}_n(\xi_0)} k \times 2\pi \int_{-1}^1 F(\mu, \theta) P_n(\mu) d\mu$$

$$= \frac{2n+1}{2\dot{Q}_n(\xi_0)} k \left[ Ak\xi_0(1-\sigma) \int_{-1}^1 \mu^2 P_n(\mu) d\mu \right.$$

$$+ A\sigma k\xi_0 \int_{-1}^1 P_n(\mu) d\mu - Bk^3\xi_0^3(1-3\sigma) \int_{-1}^1 \mu^4 P_n(\mu) d\mu$$

$$\left. - 3B\sigma k^3\xi_0^3 \int_{-1}^1 \mu^2 P_n(\mu) d\mu \right]$$

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And using Table 1 of Reference 11

$$C_0 = \frac{1}{2\dot{Q}_0(\xi_0)} k \left\{ \frac{Ak\xi_0}{3} (2+4\sigma) - Bk^3\xi_0^3 \left( \frac{2}{5} + \frac{4}{5}\sigma \right) \right\}$$

$$C_1 = 0$$

$$C_2 = \frac{5}{2\dot{Q}_2(\xi_0)} k \left\{ \frac{4}{15} Ak\xi_0 (1-\sigma) - Bk^3\xi_0^3 \left( \frac{8}{35} + \frac{4}{35}\sigma \right) \right\}$$

$$C_3 = 0$$

$$C_4 = \frac{9}{2\dot{Q}_4(\xi_0)} k \left\{ -Bk^3\xi_0^3 (1-3\sigma) - \frac{16}{315} \right\}$$

$$C_n = 0, n \geq 5$$

Therefore:

$$\begin{aligned} \phi = & \frac{1}{\dot{Q}_0(\xi_0)} k (1+2\sigma) \left[ \frac{Ak\xi_0}{3} - \frac{Bk^3\xi_0^3}{5} \right] P_0(\mu) Q_0(\xi) \\ & + \frac{1}{\dot{Q}_2(\xi_0)} k \left[ \frac{2}{3} Ak\xi_0 (1-\sigma) - \frac{2}{7} Bk^3\xi_0^3 (2+\sigma) \right] P_2(\mu) Q_2(\xi) \\ & - \frac{8}{35} \frac{1}{\dot{Q}_4(\xi_0)} k Bk^3\xi_0^3 (1-3\sigma) P_4(\mu) Q_4(\xi) \end{aligned}$$

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When the potential function of the fluid is determined, the usual practice for obtaining the mass of the entrained water is to compute the kinetic energy of the fluid and by relating this to the amplitude of motion to determine the equivalent mass. However, for this problem the distribution of the equivalent mass along the length of the hull is desired, and the amplitude of vibration is not a constant value but a function of the longitudinal position along the hull.

To avoid these difficulties the distribution of the equivalent mass is obtained by computing the pressure on the ellipsoid generated by the vibratory motion, multiplying this pressure by the projected area normal to the longitudinal axis for each unit of axial length. The equivalent mass per unit length at a given axial position is then the force per unit of length divided by the product of the amplitude of vibration at the location and the square of the circular frequency.

The pressure associated with a velocity potential function is:

$$p = \rho \left[ \frac{d\phi}{dt} + \frac{q^2}{2} + f(t) \right]$$

In this relation  $q^2$  is the square of the fluid velocity, which for the small motions is negligible.  $f(t)$  is an external pressure that does not exist.

Thus for this problem

$$p = \rho \frac{d\phi}{dt} = \rho j\omega\phi$$

since the ellipsoid is vibrating in simple harmonic motion.

Since

$$a = k\xi_0 \quad c = (\xi_0^2 - 1)^{1/2}$$

$$\xi_0 = \frac{a}{\sqrt{a^2 - c^2}} \quad k = \sqrt{a^2 - c^2}$$

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$$\begin{aligned} \frac{p}{i \omega p} &= \frac{1}{Q_0(\xi_0)} \sqrt{a^2 - c^2} (1+2\sigma) \left[ \frac{Aa}{3} - \frac{Ba^3}{5} \right] P_0(\mu) a_0(\xi_0) \\ &+ \frac{1}{Q_2(\xi_0)} \sqrt{a^2 - c^2} \left[ \frac{2}{3} Aa(1-a) - \frac{2}{7} Ba^3 \right] (2+\sigma) P_2(\mu) Q_2(\xi_0) \\ &- \frac{8}{35} \frac{1}{Q_4(\xi_0)} \sqrt{a^2 - c^2} Ba^3 (1-3\sigma) P_4(\mu) Q_4(\xi_0) \end{aligned}$$

$$\text{The axial force} = p \times \pi \left[ y^2 - \left( y + \frac{dy}{dx} \times l \right)^2 \right]$$

$$= -p \times \pi \times 2y \frac{dy}{dx} \times l$$

Since for an ellipsoid

$$y \frac{dy}{dx} = - \frac{xc^2}{a^2}$$

$$\text{The axial force} = p \left( 2\pi \frac{c^2}{a^2} x \right) \text{ lb per ft.}$$

$$\text{The entrained inertia} = \frac{2\pi p \frac{c^2}{a^2} x}{\omega^2 \delta}$$

The mathematical expression for the deflection curve which was derived earlier is :

$$\delta = a_1 x - a_3 x^3$$

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or in terms of velocity coefficients:

$$\delta = \frac{A}{1\omega} x - \frac{B}{1\omega} x^3$$

The entrained inertia therefore =

$$\begin{aligned} & - \frac{2\pi\rho \frac{c^2}{ax}}{A - Bx^2} \times \sqrt{a^2 - c^2} \left\{ \frac{(1+2\sigma)}{\dot{Q}_0(\xi_0)} \left[ \frac{A}{3} - \frac{Ba^2}{5} \right] P_0(\mu) Q_0(\xi_0) \right. \\ & + \frac{1}{\dot{Q}_2(\xi_0)} \left[ \frac{2}{3} A (1-\sigma) - \frac{2}{7} Ba^2 (2+\sigma) \right] P_2(\mu) Q_2(\xi_0) \\ & \left. - \frac{8}{35} \frac{1}{\dot{Q}_4(\xi_0)} Ba^2 (1-3\sigma) P_4(\mu) Q_4(\xi_0) \right\} \end{aligned}$$

where:

$$\mu = \frac{x}{a}, \quad Bx^2 = Ba^2 \mu^2$$

For the submarine SSB(N) 598  $a = 190.52'$   
 $c = 17.015'$

$$\xi_0^2 = \frac{(190.52)^2}{(190.52)^2 - (17.015)^2} = \frac{36,297.87}{36,008.36} = 1.00804008$$

$$\xi_0^3 = 1.012084$$

$$\xi_0 = 1.004012$$

$$\xi_0^4 = 1.0161448$$

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$$Q_0(\xi) = \frac{1}{2} \log \frac{\xi + 1}{\xi - 1}$$

$$Q_2(\xi) = \frac{1}{2} (3\xi^2 - 1) Q_0(\xi) - \frac{3}{2} \xi$$

$$Q_4(\xi) = \frac{1}{8} (35\xi^4 - 30\xi^2 + 3) Q_0(\xi) - \frac{35}{8} \xi^3 + \frac{55}{24} \xi$$

$$\begin{aligned} \dot{Q}_0(\xi) &= \frac{1}{2} \frac{\xi - 1}{\xi + 1} \frac{(\xi - 1) - (\xi + 1)}{(\xi - 1)^2} \\ &= -\frac{1}{\xi^2 - 1} \end{aligned}$$

$$\begin{aligned} \dot{Q}_2(\xi) &= \frac{1}{2} (6\xi) Q_0(\xi) + \frac{1}{2} (3\xi^2 - 1) \left( \frac{-1}{\xi^2 - 1} \right) - \frac{3}{2} \\ &= 3\xi Q_0(\xi) - \frac{3\xi^2 - 2}{\xi^2 - 1} \end{aligned}$$

$$\begin{aligned} \dot{Q}_4(\xi) &= \frac{1}{8} (35 \times 4\xi^3 - 60\xi) Q_0(\xi) - \frac{1}{8} \frac{(35\xi^4 - 30\xi^2 + 3)}{\xi^2 - 1} \\ &\quad - \frac{105}{8} \xi^2 + \frac{55}{24} \end{aligned}$$

$$\dot{Q}_4(\xi) = \left[ \frac{35}{2} \xi^3 - \frac{15}{2} \xi \right] Q_0(\xi) - \frac{1}{8} \left[ 35\xi^4 - 30\xi^2 + 3 \right]$$

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$$\begin{aligned}
 & + 105\zeta^4 - 105\zeta^2 - \frac{55}{3}\zeta^2 + \frac{55}{3} \left] \frac{1}{\zeta^2 - 1} \right. \\
 & = \left( \frac{35}{2}\zeta^3 - \frac{15}{2}\zeta \right) Q_0(\zeta) - \frac{1}{2} \left( 35\zeta^4 - \frac{115}{3}\zeta^2 + \frac{16}{3} \right) \frac{1}{\zeta^2 - 1} \\
 & = \frac{5}{2} (7\zeta^3 - 3\zeta) Q_0(\zeta) - \frac{1}{6} (105\zeta^4 - 115\zeta^2 + 16) \frac{1}{\zeta^2 - 1}
 \end{aligned}$$

$$\begin{aligned}
 Q_0(\zeta_0) &= \frac{1}{2} \log \frac{2.00401}{0.00401} = \frac{1}{2} \left[ 0.6951 - (1.3893 - 6.9078) \right] \\
 &= \frac{1}{2} (6.2136) = 3.1068
 \end{aligned}$$

$$\begin{aligned}
 Q_2(\zeta_0) &= \frac{1}{2} (3 \times 1.00804 - 1) \times 3.1068 - 3/2 (1.00401) \\
 &= 1.6382
 \end{aligned}$$

$$\begin{aligned}
 Q_4(\zeta_0) &= \frac{1}{8} \left[ 35 \times (1.00804)^2 - 30 (1.00804) + 3 \right] 3.1068 - \frac{35}{8} (1.00804)^{3/2} + \frac{55}{24} \zeta \\
 &= 3.23257 \\
 &- 4.42787 = 1.10556 \\
 &+ 2.30086
 \end{aligned}$$

$$\dot{Q}_0(\zeta_0) = - \frac{1}{\zeta^2 - 1} = - \frac{1}{0.00804008} = - 124.377$$

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$$\begin{aligned}\dot{Q}_2 (\xi_o) &= 3 \times 1.004012 \times 3.1068 - 124.377 (3 \times 1.00804 - 2) \\ &= 9.35768 - 127.377 \\ &= -118.019\end{aligned}$$

$$\begin{aligned}\dot{Q}_4 (\xi_o) &= \frac{5}{2} (7 \times 1.012084 - 3 \times 1.004012) 3.1068 \\ &\quad - \frac{1}{6} (105 \times 1.0161448 - 115 \times 1.00804008 + 16) \frac{1}{.00804008} \\ &= 31.6315 - 140.3509 \\ &= -108.7194\end{aligned}$$

$$\begin{aligned}b^2 &= (17.015)^2 = 289.51 \quad \sqrt{a^2 - c^2} = \sqrt{36,008.36} \\ &= 189.76\end{aligned}$$

$$\sigma = \text{approx. } 0.20$$

The entrained inertia =

$$\begin{aligned}& - \frac{2\pi p \times 289.51 \times 189.76}{190.52} \left\{ (1.40) \times \frac{3.1068}{-124.377} \left[ \frac{A}{3} - \frac{Ba^2}{5} \right] P_0(\mu) \right. \\ & + \frac{1.6382}{-118.019} \left[ \frac{2(0.8)}{3} A - \frac{2}{7} (2.2) Ba^2 \right] P_2(\mu) \\ & \left. - \frac{8}{35} \times \frac{1.10556}{-108.72} (1 - 0.6) Ba^2 P_4(\mu) \frac{1}{A - Ba^2 \mu^2} \right\}\end{aligned}$$

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$$\rho = 1.99 \frac{\text{lb sec}^2}{\text{ft}^4}$$

$$\frac{Ba^2}{A} = \frac{11.08 \times 10^{-12}}{4.11 \times 10^{-6}} \times (190.52)^2 = 0.0976 \text{ for 2nd mode of SSB(N) 598}$$

The entrained inertia then becomes:

$$3600 \left[ 0.01093 P_0(\mu) + 0.00655 P_2(\mu) - 0.0000976 P_4(\mu) \right]$$

$$\frac{1}{1 - 0.0976 \mu^2}$$

$$= 39.4 \left[ P_0(\mu) + 0.599 P_2(\mu) - 0.00892 P_4(\mu) \right] \frac{1}{1 - 0.0976 \mu^2}$$

The values of this entrained inertia are computed in Table B-II and plotted in Figure B-4.

#### 4. The Distribution of Entrained Water Inertia on the Actual Hull

Since the shape of the hull differs from the ellipsoid it can be expected that the inertia of the entrained water will differ from that on the idealized ellipsoid. To investigate how much difference this might be, assume that the pressure generated on the hull at any position along the longitudinal axis is the same for the actual submarine as for the ellipsoid. When this is done the distribution of water inertia is quite different as is shown in Figure B-5. It will be noted that the blunt bow and the discontinuities at the superstructure tend to give large values of water inertia and the pointed stern tends to decrease

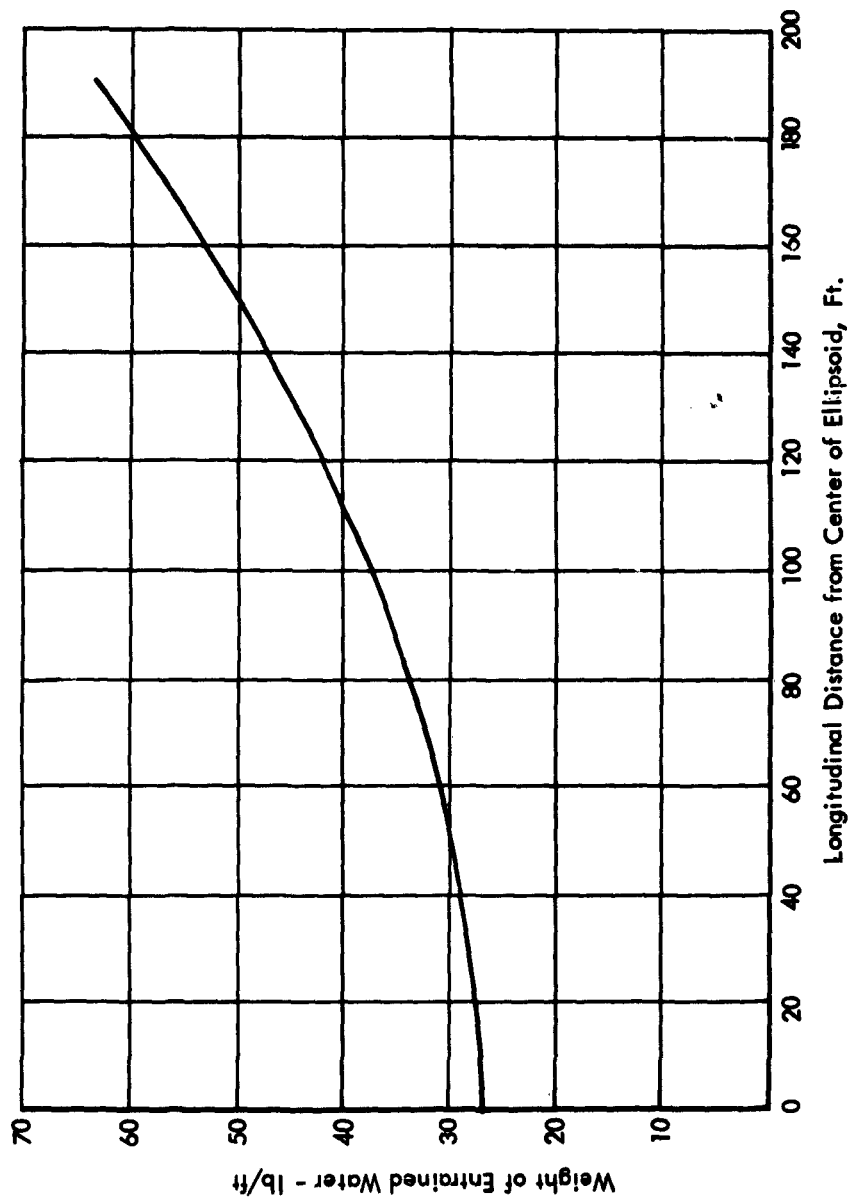


Figure B-4 - Water Inertia for an Ellipsoid representing SSB(N) 598

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TABLE B-II - CALCULATION OF LOCAL VALUES OF ENTRAINED MASS

$\frac{\Sigma}{a} = \mu$	$P_2(\mu)$	$0.599 P_2(\mu)$	$P_4(\mu)$	$-.00892 P_4(\mu)$	$\Sigma$	$\frac{1}{1-.00976\mu^2}$	M
0	-.50	-.300	0.375	-.0033	0.697	1	27.43
1	1	0.599	1	-.00892	1.590	.9902	63.30
0.5	-.125	-.0749	-.28906	.0026	0.9277	.9976	36.60
0.75	.34375	.2057	-.35010	.0031	1.2088	.9945	47.9
0.25	-.40625	-.2438	.15771	-.0014	.7548	.9994	29.78

Total Entrained Water Mass

$\mu$	Mass/ft	SM	Product
0	27.43	1	27.43
0.25	29.78	4	119.12
0.5	36.60	2	73.20
0.75	47.90	4	191.60
1.0	63.30	1	63.30
		Total	474.65

Mass =  $474.65 \times \frac{190.52}{4 \times 3} \times 2 = 15,380$  lbs. This is identical with the gross value of entrained mass used in the longitudinal vibration calculations in Reference 3.

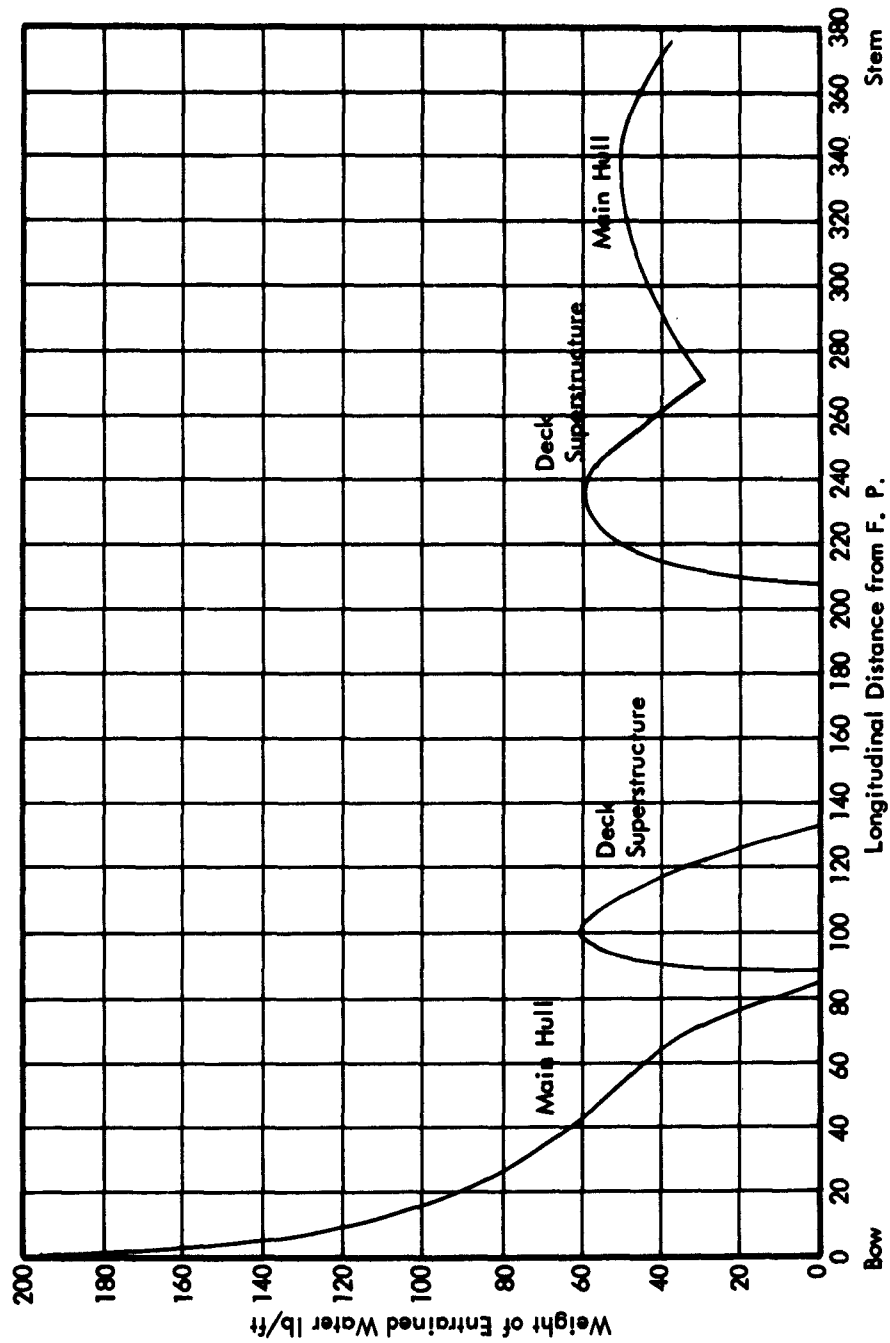


Figure B-5 - Water Inertia for SSBN 598

Using Pressures for an Ellipsoid and the Area Curve of the Actual Ship

the water inertia. Because of the large difference between the water inertia given by Figure B-5 and that of an idealized ellipsoid shown in Figure B-4, it would be highly desirable to develop an accurate method of computing the water inertia on an irregular body but the procedures for doing this are not developed and are outside of the scope of this project.

For the longitudinal vibration studies on the SSB(N) 598 the values of water inertia given in Figure B-5 are used. It should be noted that in the longitudinal modes the total mass of the entrained water is only 15,000 pounds (for either the idealized ellipsoid or the actual hull shape). This is about 1% of the submerged weight of the hull. The small value of this entrained mass contrasts strongly with the large values of entrained mass in the bending modes.

#### 5. A Brief Consideration of the Effects of Fluid Elasticity upon the Entrained Mass and Energy Dissipation of a Prolate Ellipsoid Vibrating Longitudinally

When a ship is vibrating in its bending modes it has been shown that there is no significant energy dissipated in the water. This can be explained in terms of the length of a wave length in the ship at a given vibration frequency as compared with that of sound in the water at the same frequency. It can also be shown that the entrained water carried by the ship in bending vibration can be computed accurately without considering the compressibility effects of the water. However, when a submarine is vibrating axially it is known that vibratory energy is fed into the water - explained in terms of the wave length of the axial vibration being much closer to the wave length of the sound in the water. For this reason it is desirable to have a solution for the entrained mass and for the energy dissipation of an ellipsoid vibrating longitudinally.

The solution for the vibration of a prolate ellipsoid in a compressible fluid is similar to that for the incompressible fluid but somewhat more complicated. Where the fluid is incompressible the equation for fluid potential is  $\nabla^2 \psi = 0$  but when the fluid is

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compressible the scalar wave equation  $\nabla^2 \psi = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$  applies.

For simple harmonic motion

$$\psi(x, y, z, t) = \psi(x, y, z) e^{-i\omega t}$$

and the equation simplifies to the scalar Helmholtz equation

$$\nabla^2 \psi + \beta^2 \psi = 0, \quad \beta = \omega/C \quad B(4)$$

For the submarine problem, it is desirable to express this equation in ellipsoidal coordinates  $(u, \xi, \theta)$ . These are related to the rectangular coordinates by:

$$\begin{aligned} x &= k\mu\xi & 1 \leq \xi \leq \infty \\ y &= k(1-\mu^2)^{1/2}(\xi^2-1)^{1/2} \cos \theta & -1 \leq \mu \leq 1 \\ z &= k(1-\mu^2)^{1/2}(\xi^2-1)^{1/2} \sin \theta & 0 \leq \theta \leq 2\pi \end{aligned} \quad B(5)$$

foci are at  $x = \pm \frac{1}{2} k$

Then the Helmholtz equation takes the form:

$$\begin{aligned} \frac{\partial}{\partial \mu} \left[ (1-\mu^2) \frac{\partial \psi}{\partial \mu} \right] + \frac{\partial}{\partial \xi} \left[ (\xi^2-1) \frac{\partial \psi}{\partial \xi} \right] + \frac{\xi^2-\mu^2}{(1-\mu^2)(\xi^2-1)} \frac{\partial^2 \psi}{\partial \theta^2} \\ - (\xi^2-\mu^2) \beta^2 k^2 \psi \end{aligned} \quad B(6)$$

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This may also be written in the form:

$$\frac{\partial}{\partial \mu} \left[ (1-\mu^2) \frac{\partial \psi}{\partial \mu} \right] + \frac{\partial}{\partial \xi} \left[ (\xi^2 - 1) \frac{\partial \psi}{\partial \xi} \right] + \frac{1}{(1-\mu^2)} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{(\xi^2 - 1)} \frac{\partial^2 \psi}{\partial \theta^2} + \beta^2 k^2 (\xi^2 - \mu^2) \psi = 0 \quad B(6a)$$

When we let  $\psi = \phi(\theta) S(\mu) J(\xi)$

The equation separates to the ordinary differential equations

$$\phi'' + m^2 \phi = 0 \quad B(7a)$$

$$\frac{d}{d\xi} \left[ (\xi^2 - 1) \frac{\partial J}{\partial \xi} \right] - (A - h^2 \xi^2 + \frac{m^2}{\xi^2 - 1}) J = 0 \quad B(7b)$$

$$\frac{d}{d\mu} \left[ (1-\mu^2) \frac{\partial S}{\partial \mu} \right] + (A - h^2 \mu^2 - \frac{m^2}{1-\mu^2}) S = 0 \quad B(7c)$$

where  $h = \beta k$  and  $m$  and  $A$  are separation constants.

The solutions of B(7a) are  $\phi(\theta) = \cos m\theta$   
 $\sin m\theta$

Equations B(7b) and B(7c) are identical and so the angular function  $S$  and the radial function  $J$  are solutions of the same equation over different ranges of the variable. These equations can be carried to a solution through the use of spheroidal wave functions.

Studies of the radiation of noise from submarine hulls <sup>12, 13, 14</sup> show that for the

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frequency range of propeller generated excitations, the influence of compressibility of the water is small and will tend to decrease the amount of virtual mass. Since this mass is already small, the effects of compressibility have not been explored any farther.

## APPENDIX C

### VIRTUAL INERTIA OF ENTRAINED WATER IN BENDING MODES

Since the studies of Lewis<sup>15</sup> and Taylor<sup>16</sup> on the effects of entrained water upon ship hull vibrations made practical the predictions of natural frequencies by calculations,<sup>(11, 17-23 incl)</sup> there have been only moderate advances in the understanding of water inertia. Because the complexity of the water inertia problem it has been difficult to include these advances in the calculation methods that are applied to ships.

The most practical method of including the water inertia in the calculation of ship bending modes still appears to be that used by Lewis and by Taylor. This consists of computing the water inertia per unit length of each section of the ship as though each were an infinitely long prism moving in parallel motion. The result of these calculations is a curve of sectional virtual masses. These sectional inertias are then reduced by a factor, generally designated by  $J$ , which represents the reduction in inertia that occurs because of the longitudinal shape of the hull and the modal pattern of vibration. The values of  $J$  are generally taken to be the same as those that can be computed for a translating and a vibrating ellipsoid as developed by Lamb<sup>10</sup>. Lamb<sup>10</sup> in his theoretical presentation includes a table of values of  $J$  for a translating and for a rotating rigid ellipsoid. Both Lewis and Taylor determined  $J$  for the bending ellipsoid. More recently, Macagno and Landweber<sup>11</sup> developed  $J$  values for two mode and three mode vibration. The shapes of the modal patterns were those obtained experimentally on the GOPHER MARINER<sup>24</sup> and were approximated by second and fourth order equations in the case of the two node vibrations and by third and fifth order equations in the case of the three node vibrations. The  $J$  values that are obtained for prolate ellipsoids of different ratios of major to minor axes that are vibrating in these several modes are plotted in Figure C-1. The surprising thing about this information is the change

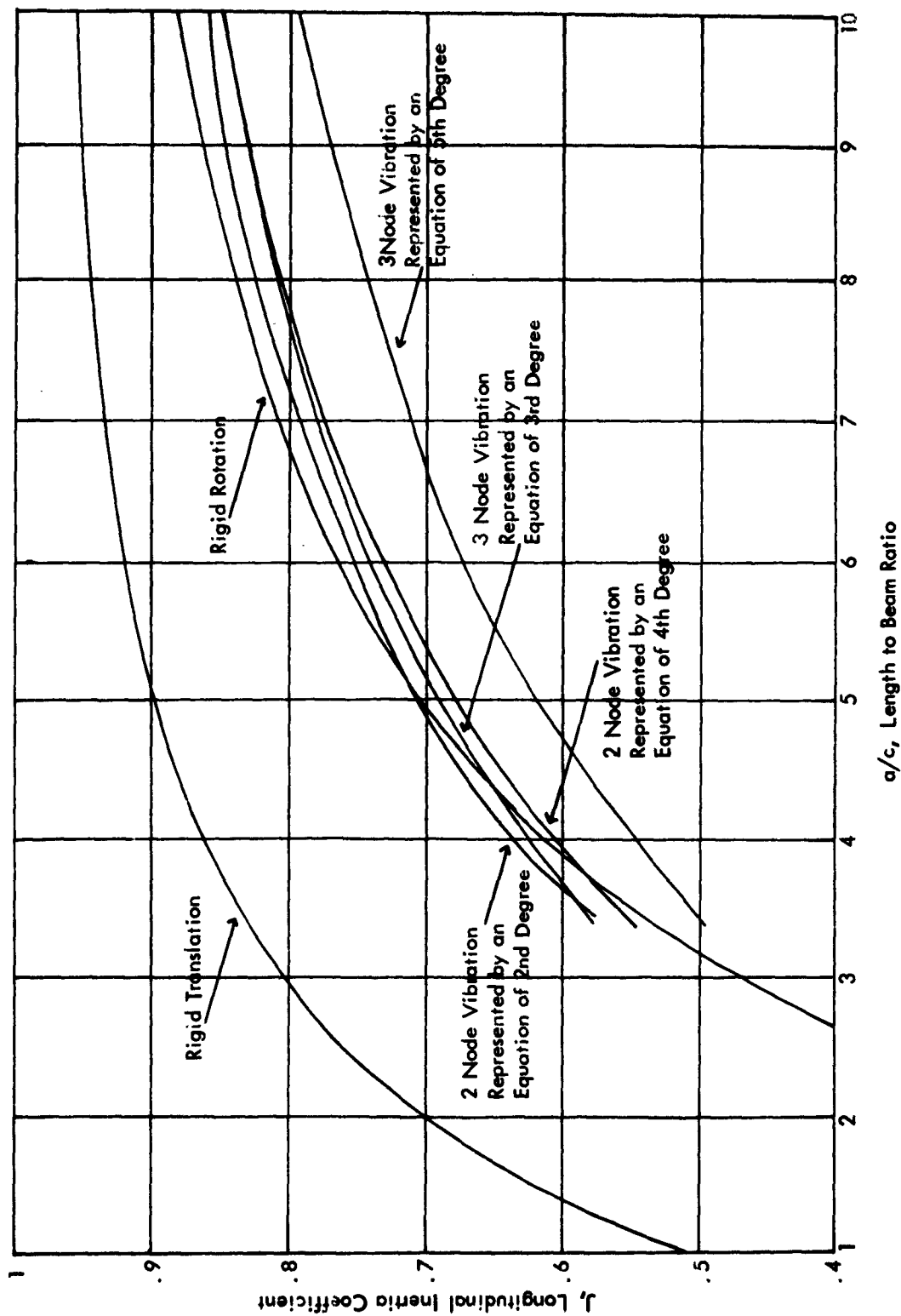


Figure C-1 - Longitudinal Inertia Coefficients for Motions of an Ellipsoid

in the value of  $J$  that occurs because of the more detailed representation of the same modal pattern as the approximating function is changed from a second to a fourth degree equation in the two node case, and by a third to a fifth degree equation in the three node case. As far as we know, there have been no determinations of  $J$  for higher modes of bending vibration than the 3 node.

The effects of multi-noded vibration patterns upon the  $J$  values are best investigated through consideration of an infinitely long circular cylinder vibrating vertically with a sinusoidal variation of amplitude along its length. This problem was first considered by Kennard<sup>25</sup> and is discussed in more detail in Appendix B of Reference 26. According to this development

$$J = - \frac{k_1(a)}{k_1'(a)} = \frac{k_1(a)}{k_1(a) - a k_0(a)}$$

where  $k_0$  and  $k_1$  are the first and second modified Bessel functions of the second kind,  $k_1'$  is the first derivative of  $k_1$  and  $a = \frac{\pi c}{N}$  where  $c$  is the radius of the cylinder and  $N$  is the distance between nodes. A plot of  $J$  as a function of  $a$  is given in Figure C-2. It is useful to have this function expressed as a simple power series in  $a$ . Within the limits  $0.6 < a < 4$

$$J \approx 0.0609 + 0.715 \frac{1}{a} - 0.19093 \frac{1}{a^2} + 0.00297 \frac{1}{a^3} + \dots$$

and within the limits  $0 < a < 0.6$   $J \approx 1.030 - \frac{4.90}{a}$

A comparison of the values of the longitudinal inertia coefficient,  $J$ , as determined for an ellipsoid by Macagno and Landweber for two noded and three noded vibrations and as determined from Kennard's relationship is given in Figure C-3. The Macagno and Landweber values are based upon a detailed study of the potential flow. It will be noted that the infinite cylinder simplification matches the theoretical value of  $J$  quite closely for the 2 node

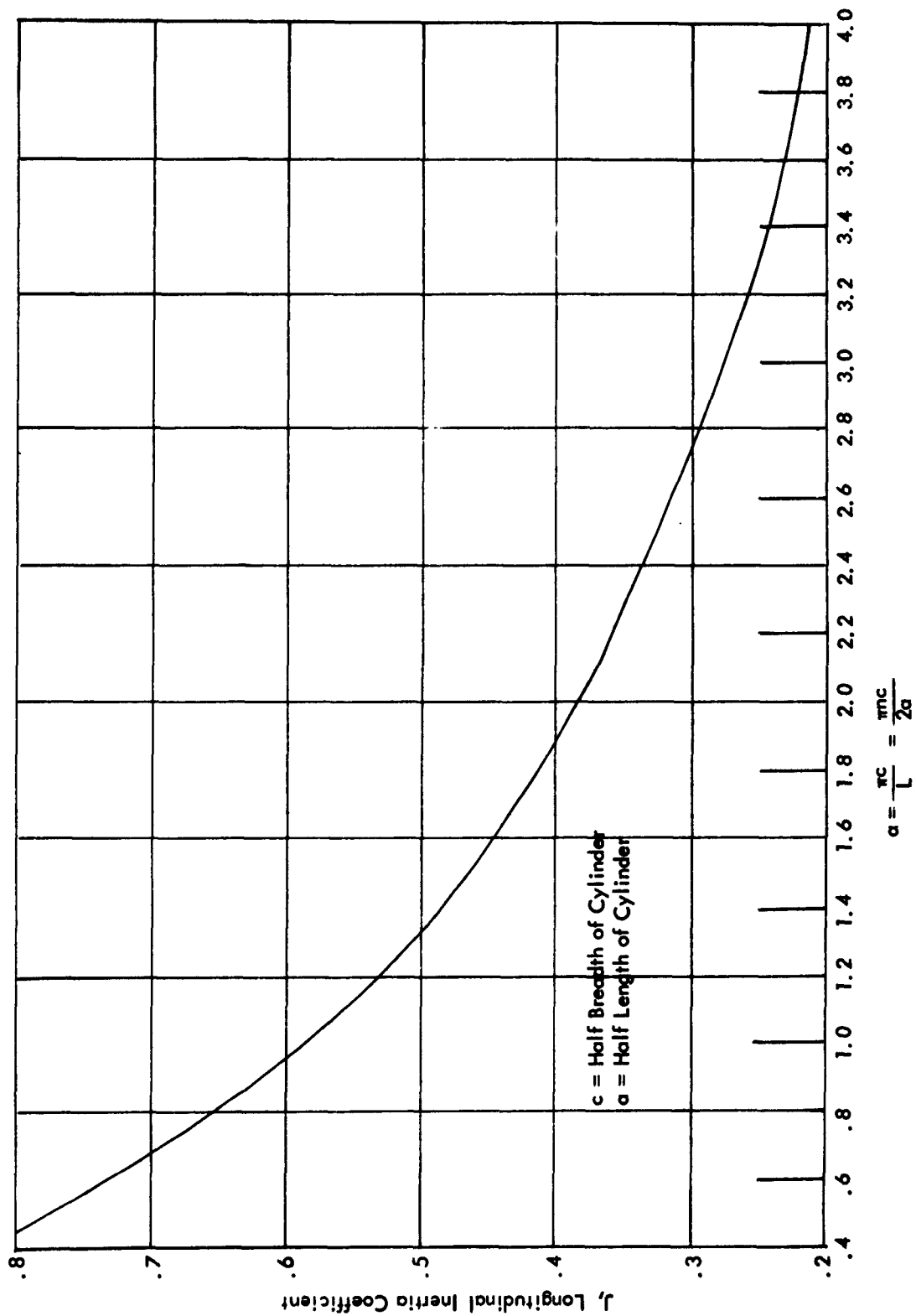


Figure C-2 - Longitudinal Inertia Coefficients for a Long Cylinder Deflecting in Sinusoidal Waves of Length  $L$

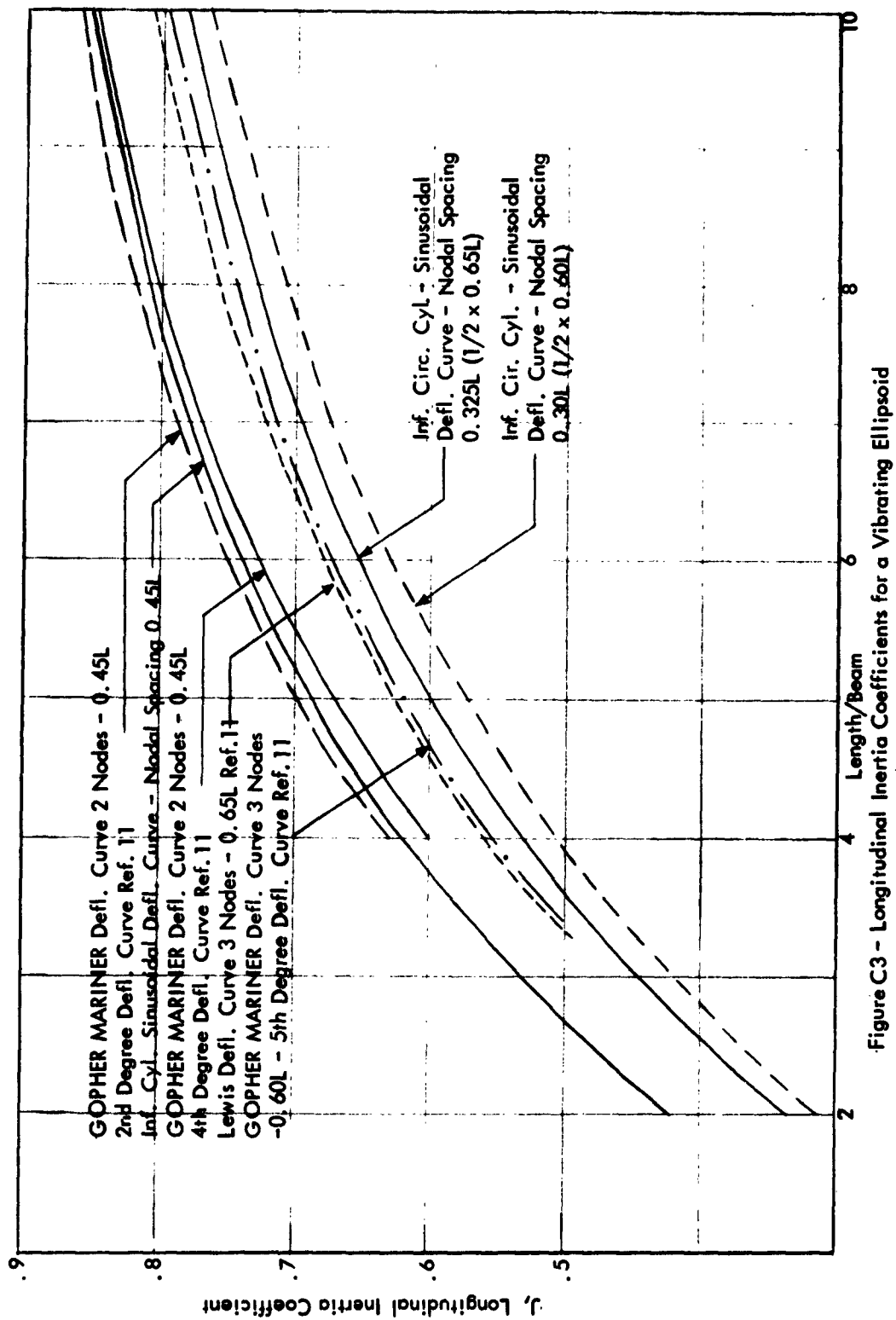


Figure C3 - Longitudinal Inertia Coefficients for a Vibrating Ellipsoid

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vibration but appears to be low (i. e., to overcorrect the water inertia) for the three node vibration. Intuitively, it would be expected that the infinite cylinder simplification would match the higher modes more accurately than the lower modes, since the end effects are of less importance.

For the present hull vibration calculations, it was decided to apply a longitudinal inertia coefficient correction to all calculations. The magnitude of this correction is taken to be 0.87 for low frequencies until in the calculation it is found that the deflection curve crosses the axis at two points. For subsequent calculations the longitudinal inertia coefficient for the calculations at any calculation frequency is taken as the value computed for the next lower frequency. The computer ascertains the distance between the most remote zero crossings and divides this distance by one more than the number of intermediate crossings to find the average distance between nodes. It then computes the value of  $a$  and the value of  $J$  for a long circular cylinder using the series formula. The water inertias used in the subsequent calculation are the section water inertias (2 dimensional values) multiplied by this determined value of longitudinal inertia coefficient,  $J$ .

## APPENDIX D

### STIFFNESSES OF THE HULL IN THE AXIAL DIRECTION, in VERTICAL BENDING AND IN VERTICAL SHEAR

The sectional stiffnesses (stiffness per unit length) of the hull are computed for the axial direction,  $AE$ ; for vertical bending,  $EI$ ; and for vertical shear,  $KAG$ , and the three values are plotted in Figure 2 along the length of the submarine. These stiffness plots reflect the discontinuities that occur as the thickness of the plating changes.

#### 1. Calculation of Axial Stiffness

The main contributor to the axial stiffness is the plating of the outer and inner shells. Since most of the sections are circular, the cross-sectional area of the steel is easy to compute. This area is corrected for the effects of slope and for the stiffening effect of the transverse framing (See Appendix E). In addition to the shell plating, longitudinal framing, keels and stringers, where continuous, contribute to the longitudinal stiffness. The contribution of such heavy longitudinal structures as pressure tanks and long heavy foundations to the longitudinal stiffness is difficult to assess. There is no adequate theory that can be used for this and so judgement is required. In Appendix F these problems are discussed and some methods developed for use in assessing the stiffness contributions of discontinuous structures.

Because of its light weight and its flexible connection to the hull, the contribution of the superstructure plating to longitudinal stiffness is neglected. Light platform decks are not included for the same reasons.

The stiffness between two stations is found by forming the summation of  $1/AE$  for the sections of constant area and finding the reciprocal of the summation.

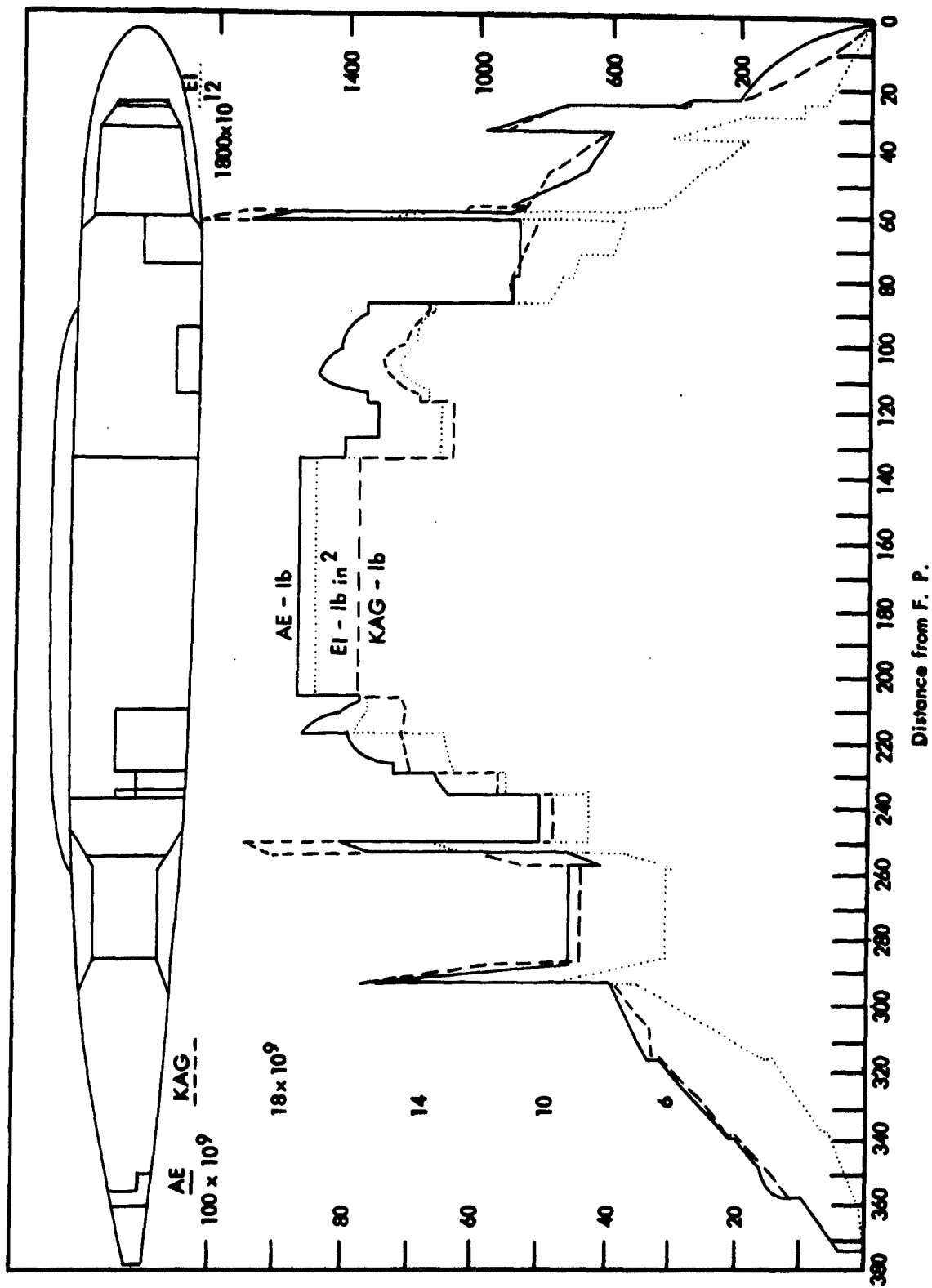


Figure 2 - SSB(N) 598 - Axial, Shear and Bending Stiffness

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**2. Calculation of Stiffness in Vertical Bending**

As with the axial stiffness, the major resistance to bending is given by the inner and outer shell plating. The stiffening effects of the frames is included in evaluating the bending stiffness but not any correction for the conical shape of the hull. Discontinuous longitudinal members are generally of less importance in bending than in axial stiffness and are treated in the same manner. The center of gravity of the section inertia is taken at the centerline of the ship. This is not entirely proper and it would probably be advisable in future calculations to compute the location of the center of the structural section relative to the centerline of the ship and to correct the moment of inertia for the offset. Figure D-2 is a computing form developed for this purpose.

The bending stiffness between two calculation stations is the reciprocal of the summation (or integral) of  $1/EI$  for sections of constant moment of inertia over the interval between the stations.

**3. Calculation of Stiffness in Vertical Shear**

The stiffness in vertical shear for a unit length ( $KAG$ ) is found by multiplying the components of the cross-sectional area by shear coefficients,  $K$ , and the shear modulus,  $G$ . Again the major elements in the shear resistance are the inner and outer shells. These elements, since they are circular shells, carry a shear coefficient of  $1/2$ . Other elements that provide shear stiffness in the vertical direction are vertical sides of tanks and foundations. For these the shear coefficient is one and is applied only to vertical webs working between heavy flanges. If these webs are discontinuous the transfer of the loading into and out of them must be estimated. Horizontal areas do not contribute to the shear stiffness.

When the overall shear stiffness between two sections is required, this is obtained from the sectional stiffnesses by forming the sum of  $1/KAG$  for the individual sections of constant  $KAG$ . The reciprocal of this summation is the shear stiffness.

Frame No.	Distance From F. P.
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100
11	110
12	120
13	130
14	140
15	150
16	160
17	170
18	180
19	190
20	200
21	210
22	220
23	230
24	240
25	250
26	260
27	270
28	280
29	290
30	300
31	310
32	320
33	330
34	340
35	350
36	360
37	370
38	380
39	390
40	400
41	410
42	420
43	430
44	440
45	450
46	460
47	470
48	480
49	490
50	500
51	510
52	520
53	530
54	540
55	550
56	560
57	570
58	580
59	590
60	600
61	610
62	620
63	630
64	640
65	650
66	660
67	670
68	680
69	690
70	700
71	710
72	720
73	730
74	740
75	750
76	760
77	770
78	780
79	790
80	800
81	810
82	820
83	830
84	840
85	850
86	860
87	870
88	880
89	890
90	900
91	910
92	920
93	930
94	940
95	950
96	960
97	970
98	980
99	990
100	1000

**Figure D-2 - Computing Form for Determining Sectional Stiffness Characteristics**

APPENDIX E

INFLUENCE OF THE CONICAL SHAPE AND OF THE TRANSVERSE  
FRAMING UPON THE STIFFNESS OF A SUBMARINE HULL

A conical hull has a somewhat lower axial stiffness per unit length than that of a cylindrical hull of the same radius and hull thickness. A hull that is stiffened by frames, because it is not free to expand and contract under Poisson's ratio effects, will be somewhat stiffer than the unsupported hull. The problem of the conical shell is considered in Reference 27 and the effects of transverse framing upon the longitudinal stiffness is mentioned in Reference 28.

Consider the conical shell reinforced by circumferential frames. A sketch of the shell is shown in Figure E1.

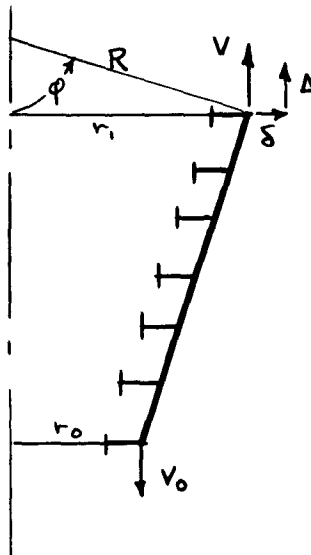


Figure E-1 - Diagram of Shell Supported by Circumferential Frames

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Assume that the effects of the frames are uniformly distributed along the length of the shell, that the frames can carry a circumferential load but no axial load and that the ratio of the frame area to the shell area per unit length is  $\beta$ . Let the pressure normal to the axis between the frames and the shell be  $P_f$ .

The following equations of equilibrium and deflection will apply (See Ref. 27).

$$V = V_0 \cdot \frac{r_0}{r} \quad \text{E1(a)}$$

$$N_x \text{ (the meridional force)} = \frac{V}{\sin \phi} \quad \text{E1(b)}$$

$$N_\theta \text{ (the hoop force)} = r P_r \quad \text{E1(c)}$$

$$\delta = r \epsilon_\theta \quad \text{E2(a)}$$

$$\Delta = -\delta \cot \phi \int_{x_0}^x + \frac{1}{\sin \phi} \int_{x_0}^x \epsilon_x dx \quad \text{E2(b)}$$

$$\text{For shell: } \epsilon_\theta = \frac{N_\theta}{Eh} - \mu \frac{N_x}{Eh} \quad \text{E3(a)}$$

$$\text{For Frames: } \epsilon_\theta = \frac{1}{\beta} \frac{N_\theta}{Eh} \quad \text{E3(b)}$$

$$\epsilon_x = \frac{N_x}{Eh} - \mu \frac{N_\theta}{Eh} \quad \text{E3(c)}$$

Equating E3(a) to E3(b) and solving for  $N_\theta$

$$N_\theta = \frac{\beta \mu N_x}{1 + \beta}$$

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$$\text{Therefore } \epsilon_x = \frac{N_x}{Eh} \left( 1 - \frac{\beta \mu^2}{1 + \beta} \right)$$

$$\delta = r \frac{\mu N_x}{(1 + \beta) Eh}$$

$$\frac{d\Delta}{dx} = \frac{\mu N_x}{(1 + \beta) Eh} \frac{dr}{dx} \cot \phi + \frac{1}{\sin \phi} \frac{N_y}{Eh} \left( \frac{1 - \beta \mu^2}{1 + \beta} \right)$$

$$= \frac{N_x}{Eh} \left[ \frac{\mu}{1 + \beta} \frac{\cos^2 \phi}{\sin \phi} + \frac{1}{\sin \phi} \left( 1 - \frac{\beta \mu^2}{1 + \beta} \right) \right]$$

$$= \frac{V}{Eh \sin^2 \phi (1 + \beta)} \left[ \mu \cos^2 \phi + 1 + \beta (1 - \mu^2) \right]$$

The unit length stiffness for an axial load is:

$$K = \frac{2\pi r h E \sin^2 \phi (1 + \beta)}{\mu \cos^2 \phi + 1 + \beta (1 - \mu^2)}$$

The correction to the area stiffness is therefore

$$\frac{\sin^2 \phi (1 + \beta)}{\mu \cos^2 \phi + 1 + \beta (1 - \mu^2)}$$

For a circular cylinder  $\phi = \frac{\pi}{2}$  and

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$$\text{the correction factor} = \frac{1 + \beta}{1 + \beta(1 - \mu^2)}$$

This agrees with Schade's value.<sup>29</sup>

The maximum correction factor will occur at a bulkhead where  $\beta$  may be considered very large and in this case will be  $\frac{1}{1 - \mu^2} = 1.1$  (about a 10% increase).

For normal framing  $\beta$  is about 0.4 and the correction factor in the cylindrical section 1.03, i. e., the stiffness is increased about 3% by the frame support.

$$\text{For an unsupported conical shape the correction factor} = \frac{\sin^2 \phi}{1 + \mu \cos^2 \phi}$$

For the steepest conical shapes on the 598 (the ends of the double hull section)  $\phi = 66.9^\circ$ . In this case the stiffness factor uncorrected for framing = 0.739 (about a 25% decrease in stiffness). For the outer surface of the hull the correction is normally only a few per cent and would only be this large near the bow of the ship.

## APPENDIX F

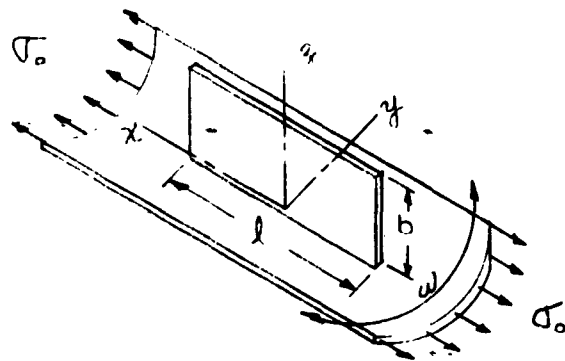
### TREATMENT OF DISCONTINUOUS STRUCTURES

Any complex structure, such as a ship, has many discontinuous structural elements in it. In the submarine that was studied the more important discontinuous elements are the tops and sides of pressure proof tanks and the tanks that are integral with the foundations.

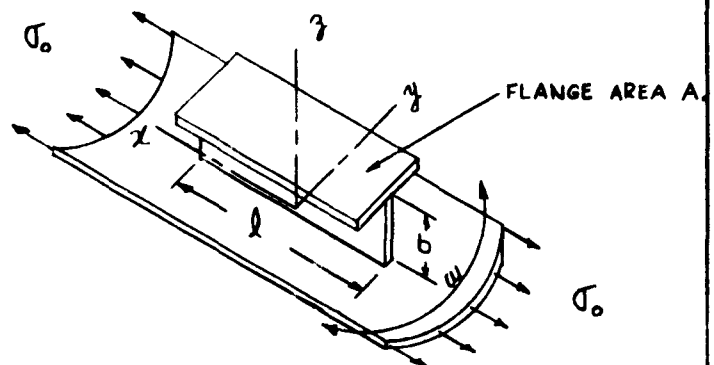
The problem can be idealized quite easily, since the hull is heavy and cylindrical bending of the hull can be neglected. The problem becomes one of the stress and strain distribution in the base plate and the stiffening plate where neither moves normal to its plane. The three examples illustrated in Figure F-1 cover all the cases necessary for the prediction of the effect of the discontinuous structure on the stiffness of the hull.

The base plate can be defined by a width  $w$  ( $= 2\pi r$  for the cylinder) and thickness  $h$ . The discontinuous plate can be characterized by a length,  $l$ , a width,  $b$ , and a thickness,  $t$ .

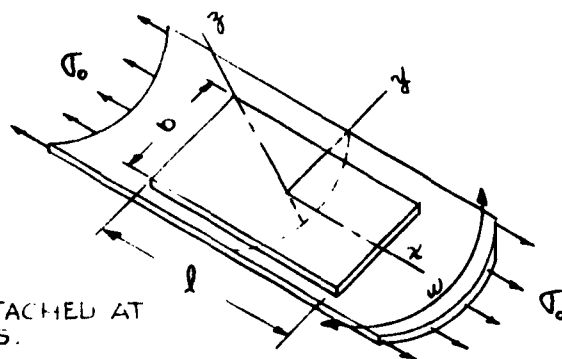
There appears to be no solution to this problem in the readily available literature and although it would be possible to develop a solution, this would require some time and is outside of the scope of this study. The results of similar studies of the effects of flange widths upon the effectiveness of beams are presented by Schade in References 29 and 30. The loading condition considered by Schade that would most closely match the problems defined above is the beam subjected to a constant moment. However, he does not develop his curves for this case. The nearest case is a beam carrying a uniform load. For a practical working procedure it was decided to utilize the curve of effective breadth ratio developed by Schade for a flange connected to double webs and carrying a uniform load. A plot of this curve (Figure 10(a) page 41 of Reference 29) is given in Figure F-2. This curve, when entered with the length of the discontinuous member,  $L$ , and the width,  $B$ , equal to the plate width when fastened on two sides or twice the plate width when fastened on one side, gives the ratio between the effective plate width and the actual plate width at the center of the plate. An application will make the process



(a) SINGLE PLATE

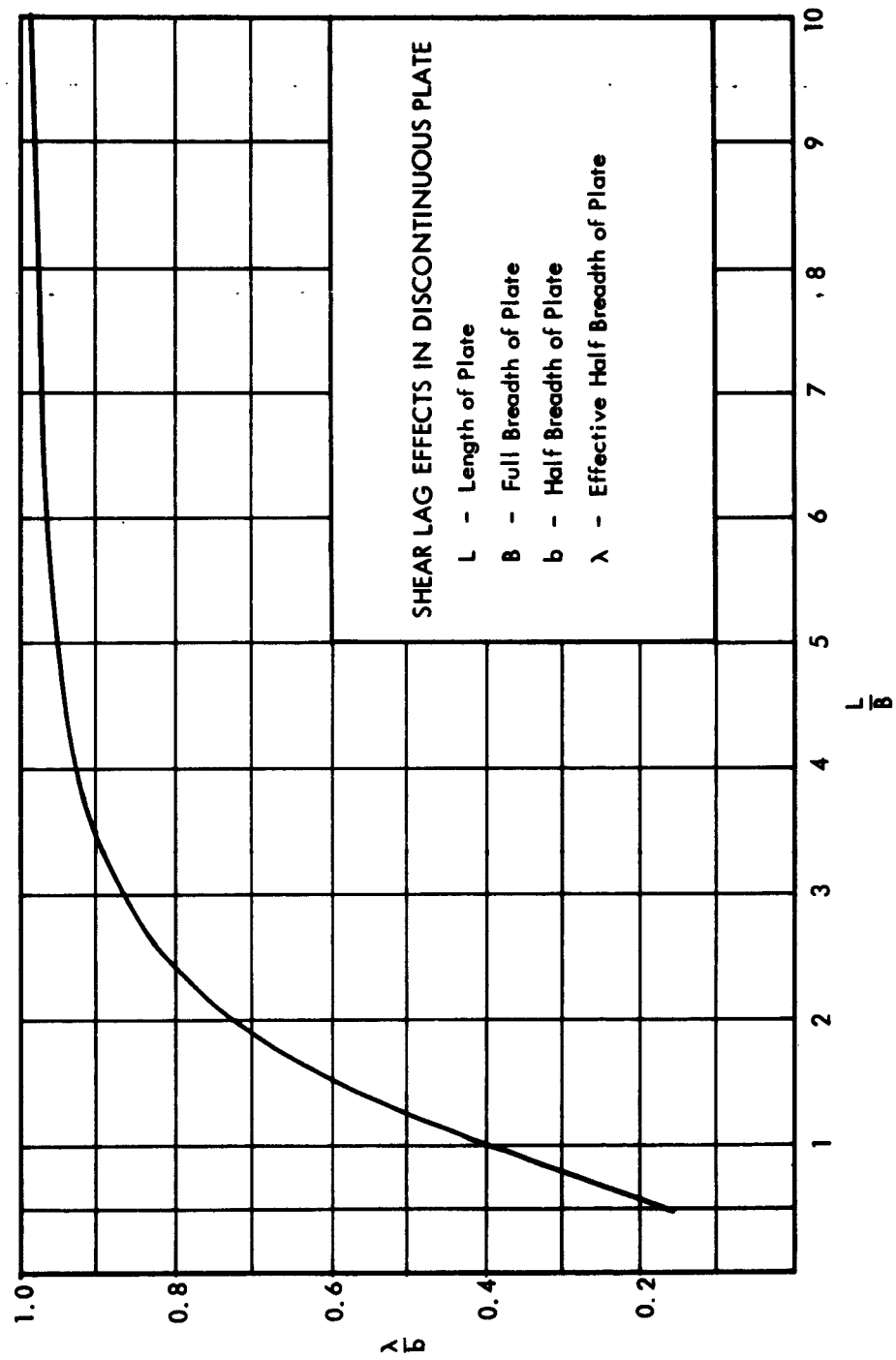


(b) PLATE WITH FLANGE



(c) PLATE ATTACHED AT TWO EDGES.

**Fig. F1** *Discontinuous plate attached to an Axially Stressed Plate that is restrained from bending.*



**Figure F-2 - Effective Breadth Ratio for Plate Attached at Both Sides**

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clear. One of the ballast tank tops in the GEORGE WASHINGTON is 20.3 feet long and 22.6 feet wide and is attached to the hull at both sides. The ratio of length to breadth is 0.90. Entering the curve for effective breadth ratio, the value of  $\lambda/b$  is found to be 0.35. Thus the effective breadth is taken as  $0.35 \times 22.6 \times 12$  or 95" and the stiffening effect of the tank top at its center would be that of an additional steel area of the hull whose area is 95" times the plate thickness. It will be noted that this stiffening effect is that at the center. As the ends of the tank are approached the stiffening effects will fall off until they become zero at the ends. The rate of fall off is assumed to be parabolic. Thus at the quarter length of the tank top the assumed equivalent breadth would be  $3/4$  of 95" or 72".

## APPENDIX G

### PROCEDURES USED IN DEFINING THE SPRUNG MASSES IN THE SHIP

One of the primary difficulties in determining the response of a ship is the definition of the structure in terms of mass and elastic constants. Normally in the past, the ship has been defined as an elastic beam to which masses representing the weights of the ship structure, cargo, and entrained water are considered to be rigidly attached. More recently, attention has been given to representing weights that are not rigidly connected to the main hull girder as masses connected to the girder through a simple spring. In applying this procedure, it is necessary to define both the main hull girder and what constitutes a rigid connection, since obviously all connections are only relatively rigid. For the purposes of the vibration calculations consider that vibrations of the ship are those of the main shear members (usually sides but sometimes also bulkheads connecting the upper and lower flanges, the deck and the bottom) and that the rigidity of the connection of all weights to these shear members, called the main hull girder, is to be investigated in determining whether a mass is sprung or not. Also for the purposes of vibration calculations any mass whose connection through the ship structure to the main hull girder is sufficiently rigid to give a natural frequency if held rigidly at the hull equal to 1.5 or more times the hull frequency that is being investigated can be considered as rigidly connected, but otherwise should be considered to be flexibly connected. Flexibly connected members can be concentrated weights (shock mounted equipment are of this category) whole portions of the ship structure or cargo and in some cases the ship bottom with its water inertia. Because of the importance of the propeller and shafting system as a flexibly mounted system, particular attention has been paid to it (See Appendix H) and a procedure developed for finding the response of elastic systems (the hull and the propulsion system) elastically connected (through the bearings).

For the submarine, fortunately, most of the structure is rigid and the only masses that are sprung are those that are shock mounted. These consist of definite machine components

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generally assembled on a common bed plate to form a package comprising a machine assembly on a common foundation that is connected to the hull through members that are highly elastic, the shock mounts. The characteristics of these mounts - both the stiffness in several directions and the damping have been accurately determined. It is thus possible to characterize these sprung-masses quite simply and accurately. In order to set a reasonable limit upon the number of masses that are treated only those masses whose weight exceeds 2000 pounds are considered. The items considered, their weights, stiffnesses and damping and location are given in Table I.

Because of the large mass of entrained water associated with them and because they are located at the stern of the ship, where the effects of mass are pronounced, the most important sprung masses are the control surfaces (stabilizers and stern diving planes) in vertical bending. The dynamic responses of these structures to a harmonic vertical motion was presented in Reference 2. For the purposes of these calculations, these surfaces are considered to be adequately represented by the sum of a rigidly attached mass and a simply sprung mass of such size that together they represent the predicted response of these structures which was presented in Reference 2. In future studies they should be incorporated in the calculations as subassemblies.

TABLE 1 - SSB(N) 598 GEORGE WASHINGTON SPRUNG WEIGHTS

Flexibly-Mounted Element	Distance from F. P. (feet)	Weight (lbs)	Longitudinal		Vertical	
			Stiffness lb/in	Damping * lb sec/in	Stiffness lb/in	Damping * lb sec/in
Each of two Fair- water diving planes	98	Long. 12,771 Vert. 7,750 37,300	Rigid		Rigid 122,200	60n
Gyro. Hyd. System	125	6,900	12,400	24.6	31,440	29.3
L. P. Blower	128	3,145	5,720	11.5	13,600	12.6
O2 Gen. Plant	213	15,200	11,600	27.8	50,000	60.7
Air Cond. Set	217	15,200	11,200	19.3	51,600	41.6
400 Cycle M. G. sets	224	12,261	8,100	15.2	38,700	33.3
H. P. Air Comp.	228	3,450	6,160	25.25	14,400	14.5
Main Coolant Pumps	251.5	51,893	307,000	192	Rigid	
Trim and Drain Pumps	268	5,300	9,000	15.9	16,000	16.4
300 KW M. G. Sets	276	32,920	20,640	107	46,000	104
H. P. Air Comp.	292	3,991	7,150	14.2	16,700	16.7
Air Cond. Sets each of 2	294	27,470	36,120	81.5	80,500	79.1
H. P. Air Comp.	298	3,991	7,150	14.2	16,700	16.7
Hydraulic Plant	346	4,300	6,400	13.6	16,800	19.8
Each of 2 stabilizers + Stern Planes	360	Long. 36,495 Vert. 68,100 125,000	Rigid		Rigid 1.820x10 <sup>6</sup>	110n

\* Damping is located between mass and foundation except for fairwater and stern planes which are to ground. Fairwater and stern plane damping constant is proportional to ship speed. This proportionality is represented by n, the frequency of oscillation, cps, for the case of a 5-bladed propeller.

## APPENDIX H

### PROCEDURES USED IN DEFINING THE MASS-ELASTIC CHARACTERISTICS OF THE PROPULSION SYSTEM

#### 1. In Longitudinal Vibration

Although the propulsion system is broken by a dental coupling, the longitudinal forces that can be carried by the coupling without causing slippage are generally larger than the longitudinal vibratory forces. The coupling therefore does not isolate the propulsion system from the propeller and shafting in longitudinal vibration. The same reasoning applies to the dental couplings in the reduction gear and the turbines. To check this, consider first the Zurn coupling located forward of the emergency propulsion motor. At 155 rpm this coupling transmits 241,000 lb ft of torque. The pitch circle of the coupling teeth is 23.50" diameter so that the total force on the teeth is 246,000 pounds. This load is carried by 94 teeth each 2-7/8" wide and therefore corresponds to a load of 910 pounds per inch on each tooth. For this type of load with turbine oil lubrication, a minimum coefficient of friction of 0.10 is reasonable. Thus the coupling could carry 25,000 pounds before slipping would occur. This is well in excess of the harmonic force across the coupling.

With the couplings considered as elastic and not isolating members the whole rotating system constitutes the vibrating system. This comprises the propeller; the thrust bearing, which is the connection to the hull; the propulsion motor, the Zurn coupling, the reduction gear, the turbines and all the connecting shafting. When the weight and longitudinal stiffness of these elements are studied they fall naturally into a number of discrete weights connected by spring elements that have low weights. The arrangement is shown in Figure 5.

The weights at the discrete points are generally formed by the summation of manufacturer's weights for the parts but in some cases, for example, the reduction bull gear, it is necessary to compute the weights of components to form another grouping. The weight

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of the propeller is increased by a water inertia for axial motion with no rotation as given by Lewis and Auslaender.<sup>31</sup> The propeller is also assumed to be a source of concentrated damping whose value is computed by the methods of Reference 31.

Proceeding from the propeller, half of the weight of the shafting between the propeller and the thrust bearing is assigned to the propeller and half to the thrust bearing mass. Also included with the thrust shaft mass is the weight of the rotor of the emergency propulsion motor which is closely connected to it. Because of the longitudinal flexibility of the web of the low speed reduction gear (the bull gear) the next natural weight division includes the bull gear shaft and hub and to this is added the Zurn disconnect coupling. The next weight item consists of the bull gear rim, the four pinions geared to it and the couplings and shafting associated with the pinions. The succeeding weight group includes the high speed gear hubs and shafts together with their couplings and shafting. The rims of the high speed gears with their mating pinions and couplings form the next weight grouping and the turbine rotors the final grouping. The weight breakdown of these items is given in Table H1.

The axial stiffnesses of the connections between the weights are generally computed from the lengths and diameters of the shafting. Where the shaft is pressed into the propeller hub and into couplings, it is assumed that the equivalent elastic length of the junction is one-third of a shaft diameter. A typical shaft stiffness calculation covering the section between the propeller and the thrust bearing is given in Table H2. A very flexible element in the longitudinal vibration system is the web connecting the hub with the rim of the low speed gear. The web consists of three plates 1" thick that connect a hub that is about 31" diameter to a rim that has an inside diameter of about 113". The web plates are connected together by 6 tubes each 19" in diameter and on a 72" diameter base circle that have a 3/8" wall thickness. The webs are bored out with 18" diameter holes in many of these tubes. They also incorporate "sound stoppers" in the form of partially flattened circumferential tubes of about 4" O.D. and 1/2" wall. Obviously the accurate calculation of the stiffness of this web would be difficult. As a guide to choosing the stiffness, we compute the stiffness

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TABLE H-1 - Weight Distribution in Propulsion System in  
Longitudinal Vibration

Station A:	
Propeller	23, 550
Entrained water	17, 730
1/2 Shafting between Propeller and Thrust Shaft	<u>10, 400</u>
Total	51, 680
Station B:	
1/2 Shafting between Propeller and Thrust Shaft	10, 400
Thrust Shaft	5, 230
Emergency Propulsion Motor and Shaft	<u>16, 270</u>
Total	31, 900
Station C:	
Zuen Coupling	3, 500
Second Reduction Gear Hub and Shaft	<u>10, 157</u>
Total	13, 657
Station D:	
Second Reduction Gear Rim and Deadening Rings	11, 010
Four pinions and Associated intermediate Shafting	<u>4, 096</u>
Total	15, 106
Station E:	
First reduction gear hub, coupling and shaft 4 at 1000 pounds each	4, 000

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TABLE H-1 (continued)

Station F:	
First reduction gear rim, web and deadening rings 4 at 828 pounds each	3,312
High speed pinions, coupling and distance 2 at 592 pounds each	<u>1,184</u>
Total	4,496
Station G:	
Turbine rotor, coupling and distance piece 2 at 3670 pounds each	7,340

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**TABLE H-2 - Typical Calculation for the Axial Stiffness  
between the Propeller and the Thrust Bearing**

Based on EB drawing 9871-00142 Bu Ships No. SSG(N) 598-203-1891801

Main Shafting Details

Section	Length	OD	ID	Area	$\frac{1}{\alpha}$
Prop. to Sleeve	6" + 6-1/4"*	16.00	4.5	185.2	0.066
Sleeve Section	75"	16.00	4.5	185.2	.405
Sleeve (bronze)	75"	17.373	16.00	9.5	15.8
Between sleeves	177-1/8"	16.00	9.00	137.5	1.289
Stuffing bx. sleeve sect.	61"	16.065	9.00	138.1	.441
Stuffing bx. sleeve (bz)	61"	17.373	16.065	8.9	13.72
Shaft	91 $\frac{13}{16}$ "	16.003	9.000	137.6	0.667
Enlargement	9-5/16" + 5.7"*	17.25	9.00	170.1	0.088
Coupling	16-1/2"	28	9	552	0.030
Thrust flange	3"	28		616	0.005
Thrust shaft	23"	15-3/4		194.8	0.118
					<u>3.088</u>

\* 1/3 of diameter for fit

+  $\frac{1}{\alpha}$  multiplied by two because of ratio of Elastic Moduli of Bronze and Steel

The shaft and its sleeve are in parallel so  $(\frac{1}{\alpha})_{\text{comb}} = \frac{1}{\alpha_{\text{shaft}}} + \frac{1}{\alpha_{\text{sleeve}}}$

The longitudinal stiffness between the propeller and the thrust collar is

$$\frac{AE}{L} = \frac{30 \times 10^6}{3.088} = 9.7 \times 10^6 \text{ lb/in.}$$

This value has been taken as  $10 \times 10^6$  in subsequent calculations.

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of a set of three webs loaded by a unit force on the rim and restrained at the center. The deflection would be that of a circular plate rigidly restrained at the inner and outer radii which is loaded at the center by a moment. The deflection of such a plate is given by Timoshenko as  $\phi = \frac{M}{a_3 E h^3}$ . Taking an inner radius of 15.5" and an outer radius of

56.5" the ratio becomes 0.275 and the corresponding value of  $a_3 = 2.7$ . Since the moment in each plate for a force  $F$  is  $\frac{F}{3} \times \frac{118}{2}$ , the deflection under load is  $\frac{118}{2} \times \phi$  and the plate thickness 1", the deflection for a unit load is  $14.3 \times 10^{-6}$  inches. The corresponding stiffness =  $0.07 \times 10^6$  pounds per inch. Because of the stiffening effects of the pipes, etc., this stiffness was arbitrarily increased to  $0.10 \times 10^6$  pounds per inch. It is not anticipated that this stiffness will be of much influence on the hull vibration and so no great efforts were expended in trying to improve its accuracy.

A very important stiffness in the response of the hull to the propeller forces is that of the connection between the shaft and the hull - i.e., the thrust bearing. For this reason the stiffness of this element was studied in considerable detail. The calculation for this stiffness presented here is based upon summing the flexibilities of the several components. The plans that are used are the following:

Kingsbury Machine Works, Inc., Philadelphia, Pennsylvania  
Kingsbury Dwg. No. 464275 - Bu Ships Dwg. No. F 3, 297, 494A  
37" Kingsbury Thrust and Journal Bearing Assembly

Kingsbury Dwg. No. 464276 - Bu Ships Dwg. No. F 3, 304, 231A - 37" (6x6) Kingsbury Thrust Bearing, Housing Details

Kingsbury Dwg. No. 464277 - Bu Ships Dwg. No. F 3, 304, 232A - 37" Kingsbury Thrust Bearing, Bearing Shell, Oil Catcher, etc. Details

Kingsbury Dwg. No. 464278, - Bu Ships Dwg. No. F 3, 303, 179A - 37" (6x6) Kingsbury Thrust Bearing, Shoe, Base Ring, Leveling Plates, etc., Details

**1. Deflection of thrust collar:**

The bending deflection is negligible

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The shear deflection is given by:

$$\delta_s = \frac{3}{4} \frac{P}{\pi h G} \log \frac{a}{r}$$

$$a = 1/2 \text{ loading diameter} = \frac{1}{2} \times 27 \frac{13}{16}$$

$$r = 1/2 \text{ root diameter} = 1/2 \times 15-3/4"$$

$$\frac{a}{r} = 1.77$$

$$h = \text{plate thickness} = 6.375"$$

$$G = \text{shear modulus} = 11.8 \times 10^6$$

$$\delta_s = 0.001812 \times 10^{-6} \text{ in/lb}$$

**2. Deflection of Spherical Surfaces under Shoes: 12" spherical radius against a flat surface  
6 shoes**

By the Hertz formula (Page 376 of Ref. 33)

$$\delta = 1.23^3 \sqrt{\frac{p^2}{E^2 R_2}} \quad p = \frac{F}{6}$$

$$\frac{\delta}{\phi p} = 1.23^3 \sqrt{\frac{1}{E^2 R_2}} \times \frac{2}{3} p^{-1/3}$$

$$= 0.82^3 \sqrt{\frac{1}{E^2 R_2 p}}$$

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assume  $F = 120,000 \text{ lb}$   $p = 20,000 \text{ R}_2 = 12''$   $E = 30 \times 10^6$

$$\text{Then } \frac{\phi \delta}{\phi p} = 0.82 \sqrt[3]{\frac{1}{(30)^2 \times 10^{12} \times 12 \times 20,000}}$$

$$= 0.137 \times 10^{-6} \text{ in/lb}$$

$$\frac{\phi \delta}{\phi F} = 0.0228 \times 10^{-6} \text{ in/lb}$$

### 3. Deflection of upper and lower leveling plate

Consider this deflection to be the sum of the deflections of two cantilevers. The first 3.8" long. 4.5" wide and 3" deep and the second 1" long. 4.5" wide and  $\frac{23}{16}$ " deep. Since there are six leveling plates which carry the load on two sides the force on each cantilever is  $F/12$ .

Therefore

$$\begin{aligned} \frac{\delta}{F} \text{ upper leveling plates} &= \frac{1}{12} \frac{(3.8)^3}{3 \times 30 \times 10^6 \times \frac{4.5 \times 3}{12}} \\ &\times \frac{1^{-3}}{3 \times 30 \times 10^6 \times 4.5 \times \frac{(\frac{23}{16})^3}{12}} = 0.00583 \times 10^{-6} \text{ in/lb} \end{aligned}$$

The lower leveling plate would have the same deflection.

### 4. Deflection in the thrust bearing housing:

Consider as direct shear between the center of the shaft and the flanges. The

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thickness of the housing is 1-3/4", the length of the housing carrying the shear is about 30 inches on each side and the length of the section across which the shear is transmitted is about 20 inches.

The deflection,  $\delta$ ,  $= \frac{F \times 20}{1.75 \times 60 \times 11.8 \times 10^6} = 0.0161 \times 10^{-6}$  in/lb. Because of bending in the housing and the flanges this is probably small. Consider the housing deflection to be  $0.03" \times 10^{-6}$ .

5. Deflection of the foundation:

The deflection of the thrust bearing foundation is computed through the use of the following plans:

EB Div. Dwg. 9841-057, Bu Ships Dwg. SSB(N) 598-112-1890616K - foundation - Propulsion Plant

EB Div. Dwg. 9832-020, Bu Ships Dwg. SSB(N) 598-101-1890537F  
Frames 83, 84, 85, 86, 87, 88, 89, 90, 91 and Tank Flat

EB Div. Dwg. 9833-006, Bu Ships Dwg. SSB(N) 598-114-189 0564C  
BHD and Framing 92, 93, 94

The thrust bearing sits on a double beam about 5 frames, i.e., 130" long. Each beam consists of upper and lower flanges about 20" wide and 3/4" thick separated by two webs each 1/2" thick. The spacing between the flanges is about 22". Therefore, the moment of inertia of each beam is

$$I = 2 \times 15 \times 11^2 \times \frac{1 \times (22)^3}{12} = 4520 \text{ in}^4$$

Assuming that these beams are pinned at the ends and are subjected to an end moment the rotation will be  $\frac{ml}{3EI}$ . Since the center of the beam is about 36" below the  $\text{CL}$  of the shaft

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the deflection at the shaft centerline for a unit load will be:

$$\frac{\delta}{P} = \frac{F \times 36 \times 130 \times 36}{3 \times 4520 \times 2 \times 30 \times 10^6} = 0.208 \times 10^{-6} \text{ in/lb}$$

The whole foundation structure is quite loosely tied except for the after end attachment to the tank flat. The deflection of this foundation structure in the longitudinal direction under a unit load can be obtained by computing the deflection of the tank top to which it is attached in shear and in bending and adding to the resulting stiffness, the additional stiffnesses of the longitudinal members.

The deflection of the tank top between frames 89 and 92 in shear is (since there are two parallel shear elements)

$$\frac{\delta}{P} = \frac{1}{2} \frac{L}{AG} = \frac{1/2 \times 50''}{3/8'' \times 78'' \times 11.8 \times 10^6} = 0.0725 \times 10^{-6} \text{ in/lb}$$

The deflection of the tank top between frames 89 and 92 in bending is determined as follows:

Because of shear lag the equivalent breadth of the tank closure on Frame 89 is obtained from Figure F-2.

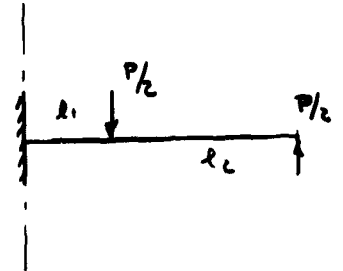
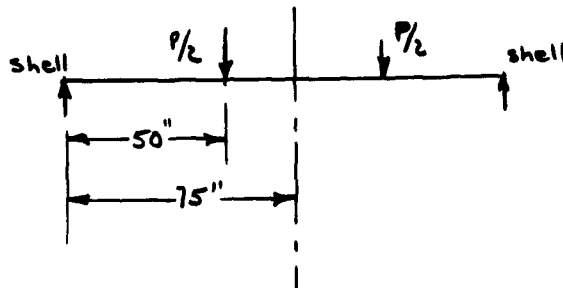
$$L = 165'', B = 54'' \quad \frac{L}{B} = 3. \quad \frac{x}{B} = 0.86$$

The central moment of inertia determined as follows:

Section	Dim.	Area	C. G.	$M_{fr}$	$I_{fr 92}$
Bhkd. Fra. 92	1-1/8" x 120"	135	0	0	
Tank Top web.	1/2 x 78"	39	39	1520	19,800
Tank closure Fr. 89	1-1/8 x 54"	61	78"	4760	371,000
		<u>235</u>	<u>26.8</u>	<u>6280</u>	<u>390,800</u>
				-	<u>169,000</u>
					= 221,800

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The beam is loaded as follows:



If a uniform beam

$$\delta = \frac{P}{2} \times l_2 \times \frac{l_1}{EI} \times l_2 + \frac{P}{2} \times \frac{l_2^3}{3EI}$$

$$= \frac{\frac{P}{2}}{30 \times 10^6 \times 222,000} \left[ 50^2 \times 25 + \frac{50^3}{3} \right]$$

$$\frac{\delta}{P} = 0.0157 \times 10^{-6}$$

The flexibility of the tank  $\frac{\delta}{P} = 92 \times 10^{-6}$  in/lb

The shear flexibility of the longitudinal members are as follows:

$$\begin{aligned} \text{For members 25" off } \frac{\delta}{P} &= \frac{60}{60 \times \frac{1}{2} \times 11.8 \times 10^6} \\ &= 0.17 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} \text{For members 19" off } \frac{\delta}{P} &= \frac{60}{40 \times \frac{1}{2} \times 11.8 \times 10^6} \\ &= 0.25 \times 10^{-6} \end{aligned}$$

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Since the tank top, the two longitudinal members 25" off the  $\mathcal{C}$  of the ship and the two longitudinal members 19" off the  $\mathcal{C}$  of the ship are in parallel, the overall flexibility of the foundation connection to the ship is the reciprocal of the sum of the reciprocals of the parts.

$$\frac{1}{\alpha_{\text{total}}} = \frac{1}{0.0882 \times 10^{-6}} + \frac{2}{0.17 \times 10^{-6}} + \frac{2}{0.25 \times 10^{-6}}$$

$$= 11.22 \times 10^6 + 11.8 \times 10^6 + 8 \times 10^6$$

$$\alpha_{\text{TOT}} = 0.0323 \times 10^{-6} \text{ in/lb}$$

When all of the flexibilities are added the total flexibility of the connection between the thrust shaft and the hull is obtained.

Element	Flexibility in/lb
Thrust Collar	$0.0018 \times 10^{-6}$
Spherical Surfaces under shoes	$0.0228 \times 10^{-6}$
Upper loading plates	$0.0058 \times 10^{-6}$
Lower leveling plates	$0.0055 \times 10^{-6}$
Thrust bearing housing	$0.0300 \times 10^{-6}$
Rotation of found. beams	$0.2080 \times 10^{-6}$
Axial defl. of foundation	$0.0323 \times 10^{-6}$
Total	$0.3065 \times 10^{-6}$

The corresponding stiffness between the thrust shaft and the hull is  $3.28 \times 10^6$ , say  $3.3 \times 10^6$  lb/in.

## 2. In Vertical Vibration

A sketch of the parts of propulsion system that participate in the vertical vibration as a separate system sprung from the hull is shown in Figure H1. It will be noted that this

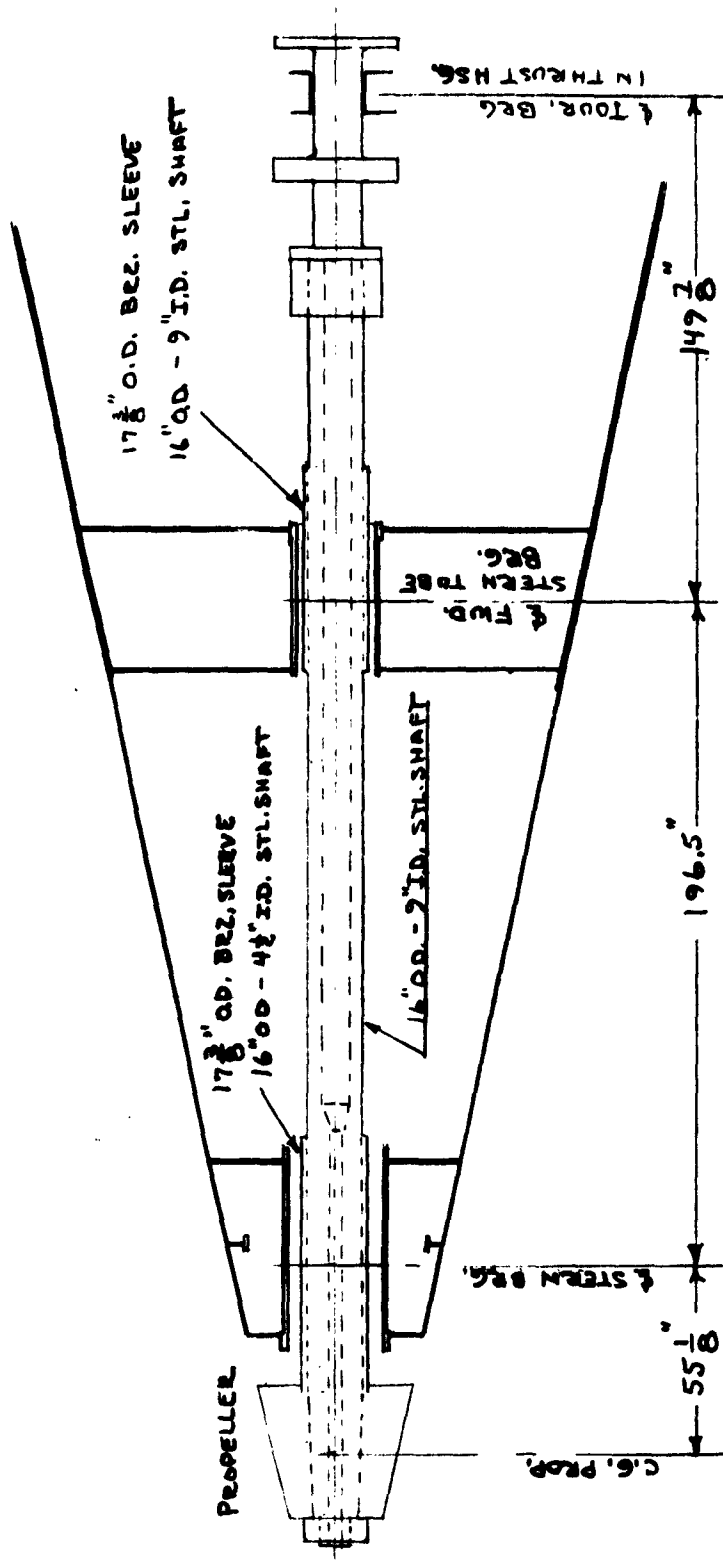


Figure H1- The Propulsion System in Vertical Vibration

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system consists of the propeller and shafting up to the thrust bearing and that the thrust bearing and the elements of the propulsion system forward of the thrust bearing are considered to be rigidly connected to the hull.

The weights at the several stations on the shaft are those of the propulsion shafting between half way to the station forward and half way to the one aft. The  $\Delta x/EI$  is formed in the same manner. The propeller weight consists of 23,550 pounds for the propeller plus 4350 pounds for the entrained water. Lewis and Auslaender<sup>31</sup> are indefinite as to the proper value to use for entrained mass in the transverse direction. The value that is shown is obtained by multiplying the value for entrained mass in the longitudinal direction with the propeller restrained from rotation, an entrained mass of 17,730 pounds, by the square of the ratio of the transverse projection of one blade at  $0.7R$  to the axial projection. This ratio  $= \text{pitch}/0.7\pi D$  and has the value 0.495. Thus the assumed value of entrained mass in the transverse direction is 4350 pounds and even this value is probably high.

The moment of inertia of the entrained water about the transverse axis is formed by applying Lewis and Auslaender's<sup>31</sup> suggestion of multiplying the value for longitudinal water inertia by  $D^2/16$  and is found to be  $40.9 \times 10^6$  pounds inches squared. The moment of inertia of the propeller itself about the transverse axis is taken a half of the polar moment of inertia and is therefore  $18.09 \times 10^6$  pounds inches squared. The sum of these is taken as  $59 \times 10^6$  pounds inches squared. The computer program does not treat the moment of inertia as concentrated with the mass but rather spread out between the concentrated masses. In accordance with this the  $59 \times 10^6$  value is considered to be distributed between the stern tube bearing and the propeller. A more precise representation of this inertia would be obtained by treating the propeller as two masses a short distance apart with the rotary inertia distributed between them.

The damping on the propeller in the transverse direction was obtained from Lewis and Auslaender<sup>31</sup> value for torsional vibration damping by the following procedures.  $b_\theta$  by Lewis and Auslaender's procedures was found to be 45,300 ft lb sec. It is assumed that the damping torque  $(=45,300\omega\theta)$  acts at  $0.7$  radius ( $0.7 \times \frac{16}{2} = 5.6$  ft). Thus the damping

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force at 0.7 radius =  $\frac{45,300}{5.6}$  can be related to the linear harmonic velocity at 0.7R,  $U_{0.7}$  ( $= 5.6\omega\theta$ ) to give the net damping force at 0.7 radius =  $\frac{45,300}{(5.6)^2} U_{0.7R} = 1440 \frac{\text{lb ft.}}{\text{sec.}}$

The force corresponding to a transverse harmonic velocity rather than a radial harmonic velocity is assumed to be half of the later, or 720  $U$ . When this value is changed to inch units and multiplied by an H factor of 0.60 the assumed value of vertical damping coefficient = 40 lb in/sec is obtained.

It remains now to determine the stiffness of the coupling between the shafting system and the hull. For the journal bearing that is incorporated in the thrust bearing housing and for all the bearings forward of this since they are oil lubricated bearings and are mounted on solid structure, the weights are assumed to be integral with the hull. The elasticities of the connections occur in the stuffing box and stern tube bearings and in bending of the shafting. The flexibilities of the bearings arise from local deformation of the hull structure, deformations of the bearing support structure and deformation in the bearing staves. When these are evaluated it is found that the local deformation of the hull is negligible, the deformation of the bearing support structure, although designed to allow some accommodation of the bearing, is small but that the deformation in the bearing staves is significant. The stuffing box is assumed to give no support to the shaft.

The flexibility of the rubber staves in the stern tube bearing is determined as follows:

The shaft diameter in the bearing is 17-3/8"

The length of the stern bearing is 60"

The static load on the stern tube bearing, taken as the propeller weight

+ 2/3 of the shaft weight, = 37,400 lbs

The projected area of the bearing = 1042 in<sup>2</sup>

The load per unit of projected area = 35.9 lb/in<sup>2</sup>

From Figure H2 the slope of the deflection curve for a No. 10 stave on a 14-5/8" diameter shaft at a load of 35.9 lb/in<sup>2</sup> is 0.001623 in/lb/in<sup>2</sup>. Thus the overall flexibility

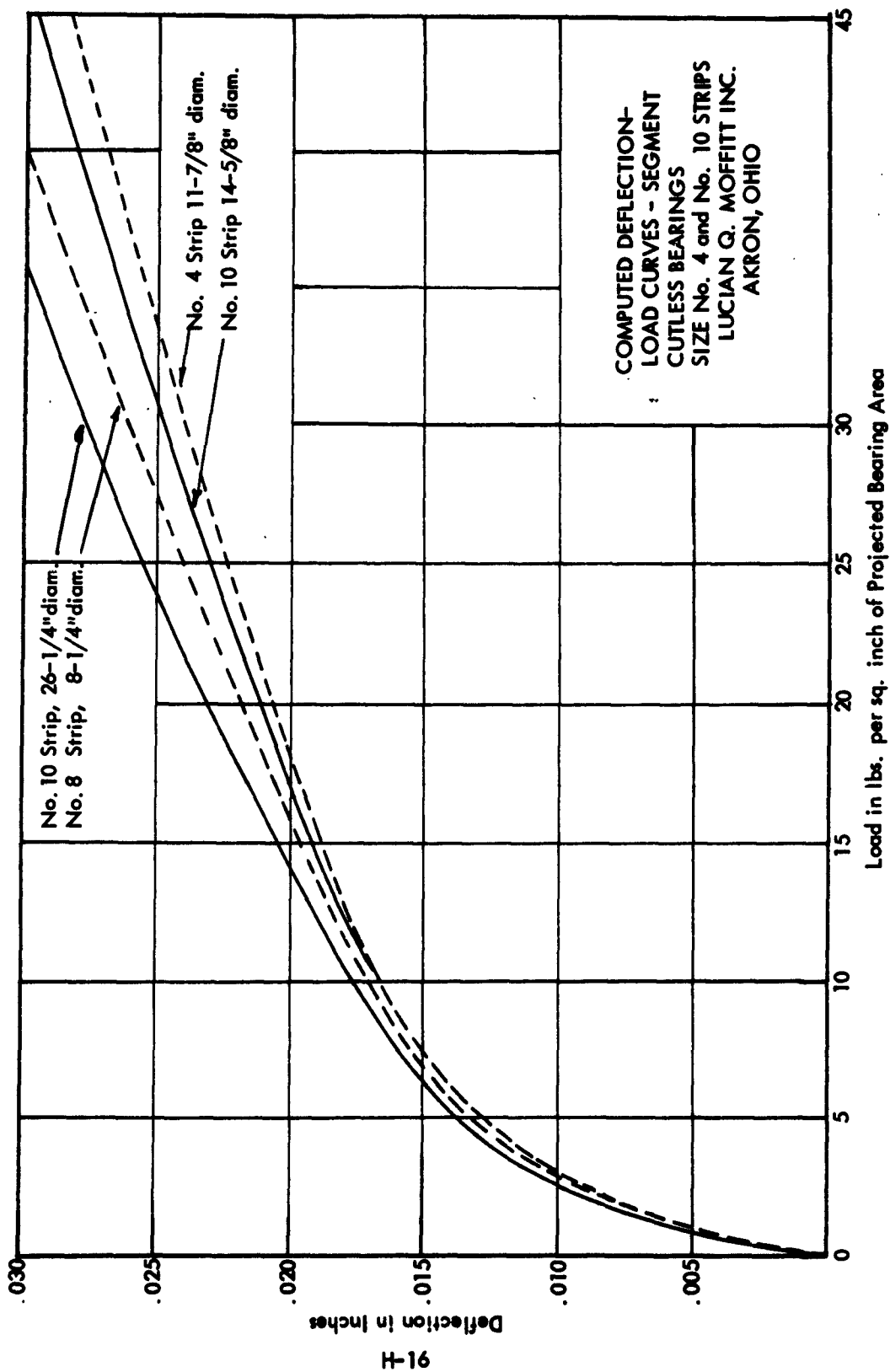


Figure H3- DEFLECTION CURVES FOR SEGMENTED CUTLESS BEARING

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of the bearing is  $0.001623 \div 1042 = 1.558 \times 10^{-6}$  in/lb. When the flexibility of the bearing support tubes, and the shaft tubes are computed they are found to be  $0.30 \times 10^{-6}$  in/lb and the corresponding stiffness of the stern tube bearing connection between shafting and hull is  $0.538 \times 10^6$  lb/in.

For the forward, stuffing box, bearing the projected area is 521 sq. in. and the load is assumed to be 7000 lb giving a pressure per square inch of projected area of  $13.5 \text{ lb/in}^2$ .

From Figure H2 the slope of the deflection curve for a No. 10 stave with a  $14\text{-}5/8$ " diameter shaft and a load of  $13.5 \text{ lb/in}^2$  is  $0.00225 \text{ in/lb/in}^2$ . Thus the overall bearing flexibility is  $0.00225/521 = 4.31 \times 10^{-6}$  in/lb.

From detailed calculations the flexibility of the forward tube and the bulkhead support is  $0.01 \times 10^{-6}$  in/lb. Thus the overall flexibility of the forward bearing is  $4.32 \times 10^{-6}$  in/lb. and the corresponding stiffness of the connection between the shafting system and the hull at this point is  $0.232 \times 10^6$  lb/in.

## APPENDIX I

### GENERALIZED BENDING RESPONSE CODE (GBRC 1)

The generalized bending response code (GBRC 1) can be used to calculate the response of beam-spring systems to specified simple harmonic driving forces and/or moments. For example, a ship hull connected elastically to other elastic systems, say a propulsion system, as well as to sprung masses, can be treated. The program is a FORTRAN II program which has been run on the IBM 7090. It is now in regular use at David Taylor Model Basin, and is referred to by number 1-840-277-01.

In this appendix, the difference equations used in GBRC 1 to represent a non-uniform beam (hull, shafting, etc.) in bending are derived. Then, these equations are modified to include more general systems such as systems of beams connected by springs. By permitting complex scaling factors for various input parameters, a variety of damping options are included, for example, Rayleigh damping and hysteresis damping. Damping coefficients for connections between sections can be given explicitly.

Longitudinal or torsional vibration problems can be solved using the program. For example, if each beam is made up of sections with spring connections only, the equations reduce to those for a mass-spring system. These equations are those used to approximate beam systems connected by springs in longitudinal or torsional vibration problems.

The program contains a special option which permits the effective mass for each section to be calculated as a function of frequency.

A detailed description of the input preparation required for GBRC 1 is included here. In Appendix J actual input sheets for the particular vertical bending calculations included in this report are given.

The output generated by GBRC 1 includes a tabulation of the input used to describe the problem, an optional summary output which lists amplitudes for deflections and moments versus frequency for a selected set of sections, and finally for each selected frequency, the

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real and imaginary parts of the deflection and moments as well as their amplitude and phase angle for each section.

**Method of Analysis**

The system of differential equations to be satisfied by the hull, shafting system, etc. in bending is given by

$$(3.1) \quad \frac{\partial V(x, t)}{\partial x} = -\mu(x) \frac{\partial^2 y(x, t)}{\partial t^2} - c(x) \frac{\partial y}{\partial t} + P(x, t)$$

$$(3.2) \quad \frac{\partial M(x, t)}{\partial x} = V(x, t) + I_{\mu z}(x) \frac{\partial^2 \gamma(x, t)}{\partial t^2} + Q(x, t)$$

$$(3.3) \quad \frac{\partial \gamma(x, t)}{\partial x} = \gamma(x, t) - \frac{V(x, t)}{KAG(x)}$$

$$(3.4) \quad \frac{\partial \gamma(x, t)}{\partial x} = \frac{M(x, t)}{EI(x)}$$

with the following notation:

- x: distance in the longitudinal direction measured from the origin of coordinates
- t: time variable
- y: displacement normal to x in the plane of bending
- $\gamma$ : angular displacement relative to the z-axis
- V: shearing force in the direction of flexural vibration (y-direction)
- M: bending moment
- $\mu$ : effective mass per unit length

- $I_{\mu z}$ : effective rotary inertia per unit length
- KAG: shear rigidity
- EI: bending rigidity
- P: external forcing function, force in y-direction
- c: damping coefficient
- Q: external forcing moment

If the forcing functions are assumed to be a simple harmonic function of time so that

$$P(x, t) = e^{i\omega t} P(x)$$

$$Q(x, t) = e^{i\omega t} Q(x)$$

Then, also,

$$y(x, t) = e^{i\omega t} y(x)$$

$$\gamma(x, t) = e^{i\omega t} \gamma(x)$$

$$V(x, t) = e^{i\omega t} V(x)$$

$$M(x, t) = e^{i\omega t} M(x)$$

and equations (1) to (4) become

$$(3.5) \quad \frac{dV(x)}{dx} = \mu(x) \omega^2 y(x) - ic(x) \omega y(x) + P(x)$$

$$(3.6) \quad \frac{dM(x)}{dx} = V(x) - I_{\mu z}(x) \omega^2 \gamma(x) + Q(x)$$

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$$(3.7) \quad \frac{dy(x)}{dx} = \gamma(x) - \frac{V(x)}{KAG(x)}$$

$$(3.8) \quad \frac{d\gamma(x)}{dx} = M(x)/EI(x)$$

Solving (3.6) and (3.7) for  $V(x)$  and  $\gamma(x)$  we obtain

$$\frac{1}{KAG(x)} \frac{dM(x)}{dx} + \frac{d\gamma(x)}{dx} = \left(1 - \frac{I_{\mu z}(x) \omega^2}{KAG(x)}\right) \gamma(x) + \frac{Q(x)}{KAG(x)}$$

or

$$(3.9) \quad \gamma(x) = \left(\frac{1 - I_{\mu z}(x) \omega^2}{KAG(x)}\right)^{-1} \left(\frac{1}{KAG(x)} \frac{dM(x)}{dx} - Q(x) + \frac{d\gamma(x)}{dx}\right)$$

and

$$\frac{dM(x)}{dx} + I_{\mu z}(x) \omega^2 \frac{d\gamma(x)}{dx} = V(x) \left(1 - \frac{I_{\mu z}(x) \omega^2}{KAG(x)}\right)$$

or

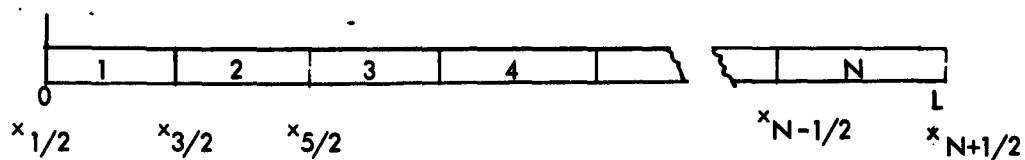
$$(3.10) \quad V(x) = \left(1 - \frac{I_{\mu z}(x) \omega^2}{KAG(x)}\right)^{-1} \left(\frac{dM(x)}{dx} + I_{\mu z}(x) \omega^2 \frac{d\gamma(x)}{dx}\right)$$

We now subdivide the interval from  $x = 0$  to  $x = L$  by the abscissas

$$x_{1/2} = 0 < x_{3/2} < x_{5/2} < \dots < x_{N+1/2} = L$$

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as shown below.



Then if (3.5) is integrated from  $x_{n-1/2}$  to  $x_{n+1/2}$ , for  $n = 1, 2, 3, \dots, N$ , we have

$$V_{n+1/2} - V_{n-1/2} = \int_{x_{n-1/2}}^{x_{n+1/2}} (\mu(x) \omega^2 - i c(x) \omega) y dx + \int_{x_{n-1/2}}^{x_{n+1/2}} P(x) dx$$

which is approximated by

$$(3.11) \quad V_{n+1/2} - V_{n-1/2} = \left[ (\mu \Delta x)_n \omega^2 - i (c \Delta x)_n \omega \right] y_n + (P \Delta x)_n$$

where

$$(\mu \Delta x)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} \mu(x) dx$$

$$(c \Delta x)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} c(x) dx$$

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$$(P \Delta x)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} P(x) dx$$

Now, from (3.10) we can approximate  $V_{n+1/2}$  for  $n = 1, \dots, N-1$  by

$$(3.12) \quad V_{n+1/2} = \frac{1}{(\Delta x)_{n,n+1}} \left( 1 - \left( \frac{I_{\mu z}}{KAG} \right)_{n,n+1} \omega^2 \right)^{-1} \left( M_{n+1} - M_n - Q_{n,n+1} \right. \\ \left. + \left( \frac{I_{\mu z}}{\mu z} \right)_{n,n+1} \omega^2 (y_{n+1} - y_n) \right)$$

where

$$Q_{n,n+1} = \int_{x_n}^{x_{n+1}} Q(x) dx$$

$$\left( \frac{I_{\mu z} \Delta x}{\mu z} \right)_{n,n+1} = \int_{x_n}^{x_{n+1}} \frac{I_{\mu z}}{\mu z} (x) dx$$

$$\left( \frac{\Delta x}{KAG} \right)_{n,n+1} = \int_{x_n}^{x_{n+1}} \frac{dx}{KAG(x)}$$

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and 
$$x_n = 1/2 (x_{n+1/2} + x_{n-1/2}) .$$

The end conditions to be imposed determine  $V_{1/2}$  and  $V_{N+1/2}$ . In particular,

(3.13) for  $V(0) = 0$  use  $V_{1/2} = 0$

for  $V(L) = 0$  use  $V_{N+1/2} = 0$

Substituting (3.12) into (3.11) and taking (3.13) into account gives for  $n = 1, 2, \dots, N$ :

(3.14)  $\alpha_{n,n+1} (M_{n+1} - M_n) + \beta_{n,n+1} (y_{n+1} - y_n)$

$$- \alpha_{n-1,n} (M_n - M_{n-1}) - \beta_{n-1,n} (y_n - y_{n-1}) = \delta_n y_n + P_n + \alpha_{n,n+1} Q_{n,n+1} - \alpha_{n-1,n} Q_{n-1,n}$$

where

$$\alpha_{n,n+1} = \begin{cases} \left[ 1 - \left( \frac{I_{\mu z} \omega^2}{KAG} \right)_{n,n+1} \right]^{-1} \frac{1}{(\Delta x)_{n,n+1}} & \text{for } n = 1, 2, 3, \dots, N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_{n,n+1} = \begin{cases} (I_{\mu z})_{n,n+1} \omega^2 \alpha_{n,n+1} & \text{for } n = 1, 2, 3, \dots, N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_n = (\mu \Delta x)_n \omega^2 - i(c \Delta x)_n \omega \quad \text{for } n = 1, 2, \dots, N$$

Now (3.8) can be integrated to give for  $n = 1, 2, \dots, N$

$$\gamma_{n+1/2} - \gamma_{n-1/2} = \int_{x_{n-1/2}}^{x_{n+1/2}} \frac{M(x)}{EI(x)} dx$$

which can be approximated by

$$(3.15) \quad \gamma_{n+1/2} - \gamma_{n-1/2} = \left( \frac{\Delta x}{EI} \right)_n M_n$$

where

$$\left( \frac{\Delta x}{EI} \right)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} \frac{dx}{EI}$$

From (3.9),  $\gamma_{n+1/2}$  can be approximated for  $n = 1, 2, \dots, N-1$

$$(3.16) \quad \gamma_{n+1/2} = \left[ 1 - \left( \frac{I_{\mu z}}{KAG} \right)_{n, n+1} \omega^2 \right]^{-1} \left[ \frac{M_{n+1} - M_n}{(KAG \Delta x)_{n, n+1}} + \frac{\gamma_{n+1} - \gamma_n}{(\Delta x)_{n, n+1}} \right]$$

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while  $\gamma_{1/2}$  and  $\gamma_{N+1/2}$  are determined by end conditions.

In particular since  $M(0) = 0$ , and  $V(0) = 0$ , we will use the end condition:

$$M_1 = 0$$

Similarly for the other end we will use

$$M_n = 0$$

Substituting (3.16) into (3.15) taking account of end conditions, we obtain for  $n = 2, 3, \dots, N-1$

$$(3.17) \quad \epsilon_{n, n+1} (M_{n+1} - M_n) - \alpha_{n, n+1} (\gamma_{n+1} - \gamma_n)$$

$$- \epsilon_{n-1, n} (M_n - M_{n-1}) - \alpha_{n-1, n} (\gamma_n - \gamma_{n-1}) = \xi_n M_n$$

where

$$\epsilon_{n, n+1} = \frac{1}{(KAG)_{n, n+1}} \alpha_{n, n+1} \quad \text{for } n = 1, 2, \dots, N-1$$

$$\xi_n = \left( \frac{\Delta x}{EI} \right)_n \quad \text{for } n = 2, 3, \dots, N-2$$

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For convenience (3. 14) and (3. 17) can be rewritten as follows:

$$\begin{aligned}
 (3. 18) \quad & -\beta_{n-1, n} y_{n-1} - \alpha_{n-1, n} M_{n-1} + (\beta_{n, n+1} + \delta_n + \beta_{n-1, n}) y_n \\
 & + (\alpha_{n, n+1} + \alpha_{n-1, n}) M_n - \beta_{n, n+1} y_{n+1} - \alpha_{n, n+1} M_{n+1} = -P_n + \alpha_{n, n+1} Q_{n, n+1} \\
 & - \alpha_{n-1, n} Q_{n-1, n}
 \end{aligned}$$

$$\begin{aligned}
 (3. 19) \quad & -\alpha_{n-1, n} y_{n-1} - \epsilon_{n-1, n} M_{n-1} + (\alpha_{n, n-1} + \alpha_{n-1, n}) y_n \\
 & + (\epsilon_{n, n+1} + \epsilon_{n-1, n} + \zeta_n) M_n - \alpha_{n, n+1} y_{n+1} - \epsilon_{n, n+1} M_{n+1} = 0
 \end{aligned}$$

For  $n = 1$  and  $n = N$ , the equations will incorporate the end conditions specified.

In summary, equations (3. 18) and (3. 19) for  $n = 1, 2, \dots, N-1$  can be rewritten in matrix notation

$$(3. 20) \quad A \vec{Z} = \vec{P}$$

where

$$(3.21) \quad A = \begin{vmatrix} A_{11} & A_{12} & 0 & \cdots & 0 \\ A_{21} & A_{22} & A_{23} & 0 & \\ 0 & A_{32} & A_{33} & A_{34} & \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & \cdots & 0 & A_{N-1, N} & \\ 0 & \cdots & 0 & A_{N, N+1} & A_{N, N} \end{vmatrix}$$

$$(3.22) \quad \vec{z} = \begin{vmatrix} \vec{z}_1 \\ \vec{z}_2 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vec{z}_N \end{vmatrix}, \quad \vec{p} = \begin{vmatrix} \vec{p}_1 \\ \vec{p}_2 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vec{p}_N \end{vmatrix}$$

where

$$(3.24) \quad \vec{z}_n = \begin{vmatrix} y_n \\ M_n \end{vmatrix}, \quad \vec{p}_n = \begin{vmatrix} \tilde{p}_n \\ 0 \end{vmatrix}$$

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$$\text{with } \tilde{P}_n = P_n - \alpha_{n-1,n} Q_{n-1,n} + \alpha_{n,n+1} Q_{n,n+1},$$

and for  $n = 2, 3, \dots, N-1$

$$A_{n,n} = \begin{vmatrix} \delta_n + \beta_{n-1,n} + \beta_{n,n+1} & \alpha_{n,n+1} + \alpha_{n-1,n} \\ \alpha_{n,n+1} + \alpha_{n-1,n} & \zeta_n + \epsilon_{n-1,n} + \epsilon_{n,n+1} \end{vmatrix}$$

$$A_{n,n+1} = \begin{vmatrix} -\beta_{n,n+1} & -\alpha_{n,n+1} \\ -\alpha_{n,n+1} & -\epsilon_{n,n+1} \end{vmatrix}$$

$$A_{n,n-1} = \begin{vmatrix} -\beta_{n-1,n} & -\alpha_{n-1,n} \\ -\alpha_{n-1,n} & -\epsilon_{n-1,n} \end{vmatrix} = A_{n-1,n}$$

and finally

$$A_{1,1} = \begin{vmatrix} \delta_1 + \beta_{1,2} & \alpha_{1,2} \\ \alpha_{1,2} & \zeta_1 + \epsilon_{1,2} \end{vmatrix}$$

$$A_{NN} = \begin{vmatrix} \delta_N + \beta_{N-1,N} & \alpha_{N-1,N} \\ \alpha_{N-1,N} & \zeta_N + \epsilon_{N-1,N} \end{vmatrix}$$

where  $\xi_1$  and  $\xi_N$  are chosen large enough so that  $M_1$  and  $M_N$  will vanish for practical purposes.

The above formulation can be generalized to apply to systems of beams with special connections. In this case each beam can be subdivided and the sections numbered as for example in Figure I-1. Then a system of matrix equations of the form

$$(3.23) \quad A \vec{z} = \vec{P}$$

can be set up where in general

$$A = \begin{vmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1N} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & A_{N3} & \cdots & A_{NN} \end{vmatrix}$$

Here the  $A_{n,m}$  are  $2 \times 2$  submatrices of  $A$  and  $\vec{z}$  and  $\vec{P}$  are defined as in (3.22). In general,  $A_{n,m}$  will contain non-zero elements whenever the section numbered  $n$  is connected to the section numbered  $m$ . In particular, if  $n$  and  $m$  are adjacent beam sections,

$$A_{n,m} = \begin{vmatrix} -\beta_{n,m} & -\alpha_{n,m} \\ -\alpha_{n,m} & -\epsilon_{n,m} \end{vmatrix}$$

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If sections n and m are connected by a spring

$$A_{n,m} = \begin{vmatrix} +k_{n,m} + i\omega c_{n,m} & 0 \\ 0 & 0 \end{vmatrix}$$

where  $k_{n,m}$  is the spring constant for the connection and  $c_{n,m}$  is the damping coefficient for the connection. In general

$$A_{n,n} = - \sum_{\substack{m=1 \\ m \neq n}} A_{n,m} + D_n$$

where

$$D_n = \begin{vmatrix} \delta_n & 0 \\ 0 & \zeta_n \end{vmatrix}$$

In addition, if section n is connected to ground,  $A_{n,n}$  must be augmented by

$$\begin{vmatrix} -k_n - i c_n \omega & 0 \\ 0 & 0 \end{vmatrix}$$

where  $k_n$  is the spring constant for the ground connection and  $c_n$  is the damping coefficient.

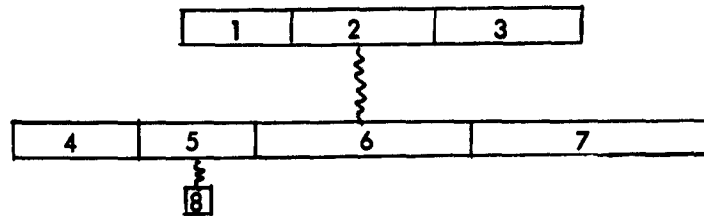


Figure I-1 - Typical Connected Beam Elements

When the above formulation is applied to the set of beams with special connections illustrated in Figure I-1 and a unit force is applied at Station 6, equation (25) takes the form

$$A\vec{z} = \vec{P}$$

where

$$A = \begin{vmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & A_{26} & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & A_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{54} & A_{55} & A_{56} & 0 & A_{58} \\ 0 & A_{62} & 0 & 0 & A_{65} & A_{66} & A_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{76} & A_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{85} & 0 & A_{88} \end{vmatrix}$$

and

$$P = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ J \\ 0 \\ 0 \end{bmatrix}, \quad J = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

### The Effective Mass

The effective mass  $(\mu \Delta x)_n$  for section  $n$ , as it is introduced in equation (3.11) and later used in the definition of  $\delta_n$  in equation (3.14), is made up of two parts. The first is the mass of the ship section. The second is a frequency dependent virtual mass which takes into account the inertial effect of the surrounding water. The method used to calculate the virtual mass is that discussed in Appendix C of this report which is based on that considered by E. K. Kennard (26).

This code provides for the automatic calculation of the virtual mass coefficient  $J$  as a function of frequency for each section  $n$  of a system and then the effective mass for section  $n$  is calculated using

$$(\mu \Delta x)_n = m_n + J(\omega) \bar{m}_n$$

where

$m_n$  is the specified mass of section  $n$

$\bar{m}_n$  is the specified water inertia associated with section  $n$

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$J(\omega)$  is the virtual mass coefficient

For each system, the displacement of each section is calculated for a set of frequencies. Until the frequency is such that the real part of the displacement for a section has at least two zeros, an initial specified value of  $J$  will be used for that system. As soon as the displacement has at least two zeros, then let  $x_a$  represent the position of the first of them, and  $x_b$  the position of the last. An average distance between the zeros of the real part of the displacement can be calculated using

$$d = \frac{x_b - x_a}{n - 1}$$

where  $n$  is the number of zeros. Then

$$J = \frac{k_1(\alpha)}{k_1(\alpha) - \alpha k_0(\alpha)}$$

where

$$\alpha = \frac{\pi c}{d}$$

$c$  is the system radius, and  $k_0$  and  $k_1$  are the first and second modified Bessel functions of the second kind.  $J$  is approximated for  $.6 < \alpha < 4$  by

$$.0609 + \frac{.715}{\alpha} - \frac{.19093}{\alpha^2} + \frac{.00297}{\alpha^3}$$

and for  $0 < \alpha < .6$  by

$$1.030 - 1.56 \frac{d}{c}$$

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### Input Preparation

The input data required for the GBRC-1 program is that required for setting up the system of difference equations described earlier. In particular, the following kinds of data can be included for each case:

- 1) The total number of beam sections. This must be less than 80.
- 2) The range of frequency values and the frequency interval for which response curves are required.
- 3) The system to which each beam section belongs, i. e., whether to the hull, shaft, or some other system.
- 4) For the hull and/or any other system, for which the J-factor is required for the added or virtual mass calculation, the following data is required:
  - a) An initial value for the J-factor
  - b) The radius
  - c) A water inertia term for each section
- 5) The mass of each section
- 6) The integral over each section of the reciprocal of the bending rigidity, i. e.,  $(\Delta x/EI)_n$ .
- 7) The integral from the mid-point of each section to the mid-point of the next of the reciprocal of the shear rigidity, i. e.,  $(\Delta x/KAG)_{n, n+1}$  or  $(\Delta x/KAG)_{n, m}$ .
- 8) The integral of the mass polar moment of inertia from the mid-point of each section to the mid-point of the next, i. e.,  $(I_{\mu z} \Delta x)_{n, n+1}$  or  $(I_{\mu z} \Delta x)_{n, m}$ .
- 9) The spring constant and damping coefficient associated with each spring connection.
- 10) For each set of parameters, for each system a complex scaling factor can be supplied. This allows damping to be introduced in various ways. In particular, a complex

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scaling factor for the ship mass corresponds to the introduction of a Rayleigh damping coefficient.

For each section and connection a system number between 1 and 80 inclusive must be specified. This number is used to select the scaling factors to be applied to the parameters for that section or connection. It is also used in order that the program can select the set of sections and connections which make up a given system so that the zeros of the displacement of that system can be determined for the calculation of the virtual mass coefficient J. Section numbers for sections which are connected to each other should differ by less than 15.

The input data cards for GBRC-1 are laid out as shown in Figures 1-2 to 1-6. The contents of these cards are described in detail below. In general, when a series of problems is being run, only those data cards which contain changes from the previous case are required. The numbers punched in columns 3 and 4 of each card identify the kind of information that card contains. An input deck should begin with a title card, and the set of data for each problem should begin with a data control card (columns 3 and 4 contain 90) which gives, for each type of data card included, the number of such cards. The data cards for the problem should follow in the same sequence as they are given on the data control card. The last card of the input deck for a set of problems should contain 99 in columns 3 and 4 if no additional problems are to be run, and 98 in columns 3 and 4 if a complete data set for another unrelated problem follows.

Detailed formats for each type of card are given below.

1) Run Title Card

<u>Columns</u>	<u>Contents</u>
7 to 12	<u>(62H<math>\Delta\Delta</math></u>
13 to 72	<u>These contain whatever title is to be</u> associated with this run.

TITLE _____		PROGRAMMER _____		DATE _____																																																																				
PROBLEM NO. _____		PHASE _____		SHEET _____ OF _____																																																																				
<div style="display: flex; justify-content: space-between; align-items: center;"> <div> <p>RUN TITLE CARD _____</p> <p>000000(62 HAA) _____</p> </div> <div> <p>DATA CONTROL CARD</p> <p>NO TYPE NO TYPE etc.</p> <table border="1" style="width: 100%; text-align: center; font-size: small;"> <tr> <td>5</td><td>7</td><td>9</td><td>11</td><td>13</td><td>15</td><td>17</td><td>19</td><td>21</td><td>23</td><td>25</td><td>27</td><td>29</td><td>31</td><td>33</td><td>35</td><td>37</td><td>39</td><td>41</td><td>43</td><td>45</td><td>47</td><td>49</td><td>51</td><td>53</td><td>55</td><td>57</td><td>59</td><td>61</td><td>63</td><td>65</td><td>67</td><td>69</td><td>71</td> </tr> <tr> <td colspan="33">0090</td> </tr> </table> </div> </div>						5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	0090																																
5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71																																							
0090																																																																								
<div style="display: flex; justify-content: space-between; align-items: center;"> <div> <p>CASE TITLE CARD</p> <p>001000(62 HAA) _____</p> </div> <div> <p>OPTION CONTROL CARD</p> <table border="1" style="width: 100%; text-align: center; font-size: small;"> <tr> <th colspan="2"></th> <th>OP1</th><th>OP2</th><th>OP3</th><th>OP4</th><th>OP5</th><th>OP6</th><th>OP7</th><th>OP8</th><th>OP9</th><th>OP10</th> </tr> <tr> <td>00200000</td> <td>9</td> <td>13</td><td>17</td><td>21</td><td>25</td><td>29</td><td>33</td><td>37</td><td>41</td><td>45</td> </tr> </table> </div> </div>								OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10	00200000	9	13	17	21	25	29	33	37	41	45																																												
		OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10																																																													
00200000	9	13	17	21	25	29	33	37	41	45																																																														
<div style="display: flex; justify-content: space-between; align-items: center;"> <div> <p>EDIT CONTROL CARD</p> <p>00210000 _____</p> </div> <div> <p>GENERAL DATA CARD</p> <table border="1" style="width: 100%; text-align: center; font-size: small;"> <tr> <th colspan="2"></th> <th><math>\omega_1</math>(CPS)</th> <th><math>\omega_2</math>(CPS)</th> <th><math>\Delta\omega</math>(CPS)</th> </tr> <tr> <td>0030</td> <td>5</td> <td>9</td> <td>17</td> <td>25</td> </tr> </table> </div> </div>								$\omega_1$ (CPS)	$\omega_2$ (CPS)	$\Delta\omega$ (CPS)	0030	5	9	17	25																																																									
		$\omega_1$ (CPS)	$\omega_2$ (CPS)	$\Delta\omega$ (CPS)																																																																				
0030	5	9	17	25																																																																				
<div style="display: flex; justify-content: space-between; align-items: center;"> <div> <p>SYSTEMS DATA CARDS</p> <table border="1" style="width: 100%; text-align: center; font-size: small;"> <tr> <th>SYSTEMS</th> <th>RADIUS</th> <th>INITIAL J</th> </tr> <tr> <td>0031</td> <td></td> <td></td> </tr> <tr> <td>0031</td> <td></td> <td></td> </tr> </table> </div> </div>						SYSTEMS	RADIUS	INITIAL J	0031			0031																																																												
SYSTEMS	RADIUS	INITIAL J																																																																						
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2) Data Control Card

<u>Columns</u>	<u>Contents</u>
3 to 4	<u>90</u> (Type of data)
5 to 6	Columns 5, 6 contain number of data cards of type given by columns 7, 8 which follow for this case.
7 to 8	
9 to 10	Similar to previous four columns for next data cards for this case.
11 to 12	
etc.	

3) Case Title Card

<u>Columns</u>	<u>Contents</u>
3 to 4	10
7 to 12	<u>62H△△</u>
13 to 72	Title for this case

4) Option Control Card

3 to 4	20
9 to 12	Op1: Added mass option selector 0 if added mass as specified in input is used, 1 if J water inertia as specified in input is to be used (See Section 4)
13 to 16	Op 2: Selector for A-matrix and P vector setup option. 0 for present.
17 to 20	Op 3: Selector for edit routine to be used at each frequency. 0 for present.
21 to 24	Op 4: Selector for final edit routine. 0 for present.

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<u>Columns</u>	<u>Contents</u>
25 to 28	Op 5: Selector for A-matrix routine. 0 if A-matrix is not printed; 1 if A-matrix is to be printed.
5) <u>Edit Control Card</u>	
3 to 4	<u>21</u>
9 to 12	Section numbers for those sections for which displacements and moments are to be tabulated vs frequency.
13 to 16	
17 to 20	
21 to 24	
25 to 28	
6) <u>General Data Card</u>	
3 to 4	<u>30</u>
7 to 8	Number of sections; this can be at most 80
9 to 16	Starting frequency in CPS
17 to 24	Upper limit for the frequency in CPS
25 to 32	Frequency interval to be used in CPS
7) <u>System Data Cards for Added Mass Calculation</u>	
3 to 4	<u>31</u>
7 to 8	System number. The hull sections, shafting sections, etc., can be distinguished from each other by assigning a system number to each set of sections. The range of these numbers can be from 1 to 80.
9 to 16	Radius associated with system in the units in which the unscaled distances between adjacent section centers are given.
17 to 24	Initial J-value for use with this system.

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8) Section Parameter Cards

<u>Columns</u>	<u>Contents</u>
3 to 4	43 or 44
7 to 8	Section number
12	End condition - normally 0. 1 for $V(o) = M(o) = 0$ 2 for $V(L) = M(L) = 0$
15 to 16	System number
17 to 24	Ship mass
25 to 32	Water inertia
33 to 40	$(\Delta x/EI)_n$
41 to 48	$(\Delta x)_{n, n+1}$
49 to 56	$(\Delta x/KAG)_{n, n+1}$
57 to 64	$(I_{\mu z} \Delta x)_{n, n+1}$
65 to 72	$P_n$ if columns 3 and 4 contain 43, $Q_{n, n+1}$ if those columns contain 44.

9) Scaling Factor Cards for Section Parameters

<u>Columns</u>	<u>Contents</u>
3 to 4	41 if this card contains real parts of scaling factor. 42 if this card contains the imaginary parts of the scaling factors.
15 to 16	System number for which scaling factors on this card are to be applied.
17 to 72	Scaling factor for each parameter are given in those columns containing that parameter value on the section parameter cards.

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10) Special Connection Parameter Cards

3 to 4	<u>53</u>
7 to 8) 11 to 12)	The section numbers for the connection - n and m if m = 0, this will represent a connection from section n to ground.
15 to 16	System number with which connection is to be associated
17 to 24	$k_{n,m}$ Spring constant for connection
25 to 32	$C_{n,m}$ Constant damping coefficient for connection
33 to 40	$C_{n,m}/\omega$ Coefficient for frequency dependent damping for connection
41 to 48	$(\Delta x)_{n,m}$
49 to 56	$(\Delta x/KAG)_{n,m}$
57 to 63	$(I_{\mu z} \Delta x)_{n,m}$

Note that in general either  $k_{n,m}$ ,  $C_{n,m}$  and/or  $C_{n,m}/\omega$  or  $(\Delta x)_{n,m}$ ,  $\frac{\Delta x}{KAG}_{n,m}$

and  $(I_{\mu z} \Delta x)_{n,m}$  will be supplied depending on whether sections n and m are connected by a spring or are two adjacent sections of a beam. It is important that the system numbers in these two cases be distinct, if J factors for the virtual mass calculation are to be calculated for the beam.

11) Scaling factor cards for parameters in special connections

<u>Columns</u>	<u>Contents</u>
3 to 4	51: This card contains real parts of scaling factors 52: This card contains imaginary parts of scaling factors.

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	<u>Columns</u>	<u>Contents</u>
	13 to 16	System number with which these scaling factors are to be associated
	17 to 56	Scaling factors for parameters in corresponding locations on special connection Parameter Cards.
12) <u>End of Data</u>	3 to 4	99

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APPENDIX J

INPUT DATA FOR VERTICAL BENDING CALCULATIONS



TITLE \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE 10/10  
 PROBLEM NO. Case 12 4-962 PHASE \_\_\_\_\_ LABEL 84D 277 SHEET 2 OF 3

# SECTION DATA CARDS

SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_n, n+1$	$(\Delta x/KAG)_{n,n+1}$	$(\Delta x \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	1.0	1.0	1.0E-8	1.0	1.0E-5	1.	1.
00410000									
00410000									

# IMAGINARY PART OF SCALING FACTORS

00420000									
00420000									
00420000									

# PARAMETER VALUES - UNSCALED

00430001	0001	2.3520	5.4795	12.8232	15	2.1319	62.44		
00430002	0001	2.5605	4.4091	8.0585	15	.9204			
00430003	0001	2.0220	2.8665	3.7922	15	.6671			
00430004	0001	4.8915	4.8925	2.0146	15	.5414			
00430005	0001	4.8300	5.4045	1.4015	15	.4642			
00430006	0001	5.0520	6.4470	1.1115	15	.3439			
00430007	0001	8.8680	7.3140	1.0233	15	.3254			
00430008	0001	8.9085	8.0610	0.9343	15	.3182			
00430009	0001	11.7810	8.5635	0.5655	15	.3324			
00430010	0001	12.4485	8.9340	0.4327	15	.2494			
00430011	0001	15.8820	9.4170	0.3663	15	.2084			
00430012	0001	11.7195	9.7815	0.3358	15	.3005			
00430013	0001	11.0325	9.8055	0.3328	15	.2005			
00430014	0001	11.7615	9.8055	0.3358	15	.2005			
00430015	0001	11.8020	9.8055	0.3358	15	.2005			

[illegible]

TITLE <u>CARC 1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>Cornesco Case 13</u>		PHASE <u>4-962</u>		SHEET <u>277</u> OF <u>8</u>	
LABEL <u>890</u>					

RUN TITLE CARD		<u>SSB(N) 598</u>		<u>OCT 20, 1962</u>		<u>Cornesco</u>		<u>Vertical</u>		<u>Banding</u>																								
DATA CONTROL CARD																																		
NO TYPE NO TYPE etc.																																		
5	7	3	5	15	7	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71		
000001	10	01	21	01	30	04	41	01	42	44	43	02	57	01	22	18	53																	
CASE TITLE CARD																																		
001000 (62 HAA Case 13		<u>25 Masses in Hull</u>										<u><math>\eta = .87</math> For <math>1\text{ to }3.5\text{ cps}</math></u>																						
OPTION CONTROL CARD																																		
00200000		001	002	003	004	005	006	007	008	009	010																							
		3	17	21	25	29	33	37	41	45																								
EDIT CONTROL CARD																																		
00210000		0001	0041																															
		13	17	21	25	29																												
GENERAL DATA CARD																																		
0030		42	1.0	3.5	0.5																													
		5	17	25																														
SYSTEMS DATA CARDS																																		
SYSTEMS		RADIUS										INITIAL J																						
0031																																		
0031																																		

TITLE \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_

PROBLEM NO. Case 13 PHASE \_\_\_\_\_ LABEL 890 217 SHEET 2 OF 2

### SECTION DATA CARDS

SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(IHz \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	0022591	002250	0.990E-12	12	0.990E-4		1.0
00410000		2	0022591						
00410000									

### IMAGINARY PART OF SCALING FACTORS

00420000					-0.990E-12		-0.990E-9		
00420000									
00420000									

### PARAMETER VALUES - UNSCALED

0043 0001	0001	200 000	133 300	10-00	15		0.2		
0043 2	1	519 000	126 000	1.157	15		24.0		
0043 3	1	660 000	276 000	0.342	15		19.1		
0043 4	1	921 000	631 000	0.243	15		13.2		
0043 5	1	634 000	764 000	0.164	15		11.03		
0043 6	1	449 000	744 000	0.176	15		13.33		
0043 7	1	810 400	840 000	0.128	0.		0.		
0043 0010	1	765 000	902 000	0.127	15		12.6		
0043 11	1	973 100	425 000	0.128	0.		0.		
0043 14	1	883 700	429 000	0.101	15		11.20		
0043 15	1	492 200	930 000	0.107	12		11.20		
0043 16	1	442 000	930 000	0.107	12		11.20		
0043 17	1	478 000	930 000	0.107	12		11.20		
0043 18	1	421 800	424 000	0.110	12		12.38		
0043 19	1	1072 200	900 000	0.130	0.		0.		

TITLE \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 13 PHASE \_\_\_\_\_ LABEL 840 SHEET 3 OF 3

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{H2A})_{n,n+1}$	$P_n$	
0043	23		0001	1 156 200	842 000	0.175	0.	0.			
0043	23			779 043	794 000	0.211	15	16.92			
0043	26			620 900	694 000	0.343	0.	0.			
0043	24			257 300	612 000	0.352	15	18.43			
0043	30			390 850	557 000	0.324	0.	0.			
0043	35			354 000	452 000	0.454	15	28.63			
0043	36			280 100	354 000	0.825	15	36.3			
0043	37			214 300	249 000	1.440	15	52.4			
0043	38			209 700	232 200	4.046					
0043	41	0002		191 700	48 900	10				1.0	
0043	5		2	37 300							
0043	4		2	37 300							
0043	12		2	6 900							
0043	13		2	3 145							
0043	20		2	15 200							
0043	21		2	15 200							
0043	22		2	12 261							
0043	24		2	3 420							
0043	27		2	5 300							
0043	28		2	32 920							
0043	31		2	3 991							
0043	32		2	27 470							
0043	33		2	27 470							
0043	34		2	3 991							



TITLE _____		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>CORR-00 Case 13</u>		PHASE _____		SHEET <u>5</u> OF <u>8</u>	
LABEL <span style="border: 1px solid black; padding: 2px;">840</span>		277			

5	9	13	17	25	33	41	49	57	65	73

SPECIAL CONNECTION CARDS						
n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$
<b>REAL PART OF SCALING FACTORS</b>						
0051		2001				1.2
0051		2	1.0	1.0	1.0	0.990E-9
0051						
<b>IMAGINARY PART OF SCALING FACTORS</b>						
0052		0001				-0.099E-9
0052						
0052						

PARAMETER VALUES - UNSCALED						
0033	2007	0008	0002	122 200	0.	
0033	7	7	2	122 200	0.	
0033	0011	0012	2	31 440	25.3	
0033	11	13	2	13 600	12.6	
0033	19	20	2	50 000	60.7	
0033	19	21	2	57 600	41.6	
0033	19	22	2	38 700	33.8	
0033	23	24	2	14 400	14.5	
0033	26	27	2	16 000	16.4	
0033	26	28	2	46 000	18.4	

TITLE \_\_\_\_\_ CASE 13 \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. \_\_\_\_\_ PHASE \_\_\_\_\_ LABEL 840 277 SHEET 4 OF 8

SPECIAL CONNECTIONS - PARAMETER VALUES										25	33	41	49	57	65	73
	N	M	SYSTEM	K <sub>n,m</sub>	C <sub>n,m</sub>	C <sub>n,m</sub> /ω	(Δx) <sub>n,m</sub>	(Δx/KAG) <sub>n,m</sub>	(1/42 Δx) <sub>n,m</sub>							
0053	20	0031	0001	16700	16.7											
0053	30	32	2	80500	79.1											
0053	36	34	2	80500	79.1											
0053	30	34	2	14700	16.7											
0053	35	39	2	14800	19.8											
0053	33	40	2	1820000												
0053	41	42	2	1820000												
0053	7	11	1				15	12.23								
0053	11	14	1				15	12.38								
0053	14	23	1				12	13.65								
0053	23	42	1				15	18.34								
0053	26	29	1				15	20.75								
0053	30	32	1				15	25.22								
0053	38	41	1				15	108.5								
0053	3		2			60.										
0053	5		2			60.										
0053	40		2			110.										
0053	42		2			110.										
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TITLE \_\_\_\_\_  
 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Coneco Case 13 PHASE \_\_\_\_\_ LABEL 840 SHEET 5 OF 8

## SECTION DATA CARDS

[illegible]

**J-12**

TITLE GBRC 1: DATA FORMAT 2 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 14 PHASE \_\_\_\_\_ LABEL 890 277 SHEET 2 OF 2

SECTION DATA CARDS

5 9 13 17 25 33 41 49 57 65 73

SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(Hz \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	00000000	00000000	0.990E-12	1.0	0.990E-9		1.0
00410000		0003	00000000		0.990E-12	1.0	0.990E-9	2591.0	1.0
00410000									
IMAGINARY PART OF SCALING FACTORS									
00420000		0003			-0.990E-12		-0.990E-9		
00420000									
00420000									
PARAMETER VALUES - UNSCALED									
0043 0043		0003	13400.		1214	196.5			
0043 0044		0003	8220.		641.5	55.125		59.0	
0043 0045		0003	17900.		75.5				
0043									
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TITLE <u>GBRC1: DATA FORMAT 3</u> PROGRAMMER _____ DATE _____									
PROBLEM NO. <u>Case 14</u> LABEL <u>81710</u> PHASE _____ SHEET <u>3</u> OF <u>5</u>									

5	0	15	17	25	33	41	49	57	65	73

SPECIAL CONNECTION CARDS									
n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{HIZ} \Delta X)_{n,m}$	
REAL PART OF SCALING FACTORS									
0051			0003	1.			1.0	0.990 E-9	
0051									
0051									
IMAGINARY PART OF SCALING FACTORS									
0052			0003					-0.999 E-9	
0052									
0052									

PARAMETER VALUES - UNSCALED									
0053	37	48000.3				149.875			
0053	38	48000.2	232000.						
0053	41	44000.2	538000.						
0053	45	0002	40.						
0053									
0053									
0053									
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0053									

TITLE <u>GBRC.1: DATA FORMAT 1</u>															PROGRAMMER _____										DATE _____																																																																																
PROBLEM NO. <u>Case 14</u>															PHASE _____										SHEET <u>4</u> OF <u>5</u>																																																																																
															LABEL <u>840</u> <u>277</u>																																																																																										
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<table border="1" style="width:100%; border-collapse: collapse; text-align: center;"> <tr> <td>5</td><td>7</td><td>3</td><td>13</td><td>15</td><td>7</td><td>19</td><td>21</td><td>23</td><td>25</td><td>27</td><td>29</td><td>31</td><td>33</td><td>35</td><td>37</td><td>39</td><td>41</td><td>43</td><td>45</td><td>47</td><td>49</td><td>51</td><td>53</td><td>55</td><td>57</td><td>59</td><td>61</td><td>63</td><td>65</td><td>67</td><td>69</td><td>71</td> </tr> <tr> <td>0090</td><td>01</td><td>10</td><td>01</td><td>30</td><td>01</td><td>41</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> </table>																																								5	7	3	13	15	7	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	0090	01	10	01	30	01	41																										
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0090	01	10	01	30	01	41																																																																																																			
CASE TITLE CARD _____																																																																																																									
001000 (62 HAA Case 14) <u>U=82 For 8.5 To 25.0 cps</u>																																																																																																									
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<table border="1" style="width:100%; border-collapse: collapse; text-align: center;"> <tr> <td>20</td><td>21</td><td>OP2</td><td>OP3</td><td>OP4</td><td>OP5</td><td>OP6</td><td>OP7</td><td>OP8</td><td>OP9</td><td>OP10</td> </tr> <tr> <td>00200000</td><td>3</td><td>17</td><td>21</td><td>25</td><td>29</td><td>33</td><td>37</td><td>41</td><td>45</td><td></td> </tr> </table>																																								20	21	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10	00200000	3	17	21	25	29	33	37	41	45																																													
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<table border="1" style="width:100%; border-collapse: collapse; text-align: center;"> <tr> <td colspan="2">No. of</td> <td><math>\omega_1</math> (CPS)</td> <td><math>\omega_2</math> (CPS)</td> <td><math>\Delta\omega</math> (CPS)</td> </tr> <tr> <td>0030</td> <td>0045</td> <td>8.5</td> <td>25</td> <td>0.5</td> </tr> <tr> <td>5</td> <td>3</td> <td>17</td> <td>25</td> <td></td> </tr> </table>																																								No. of		$\omega_1$ (CPS)	$\omega_2$ (CPS)	$\Delta\omega$ (CPS)	0030	0045	8.5	25	0.5	5	3	17	25																																																				
No. of		$\omega_1$ (CPS)	$\omega_2$ (CPS)	$\Delta\omega$ (CPS)																																																																																																					
0030	0045	8.5	25	0.5																																																																																																					
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<table border="1" style="width:100%; border-collapse: collapse; text-align: center;"> <tr> <td>SYSTEMS</td> <td>RADIUS</td> <td>INITIAL J</td> </tr> <tr> <td>0031</td> <td></td> <td></td> </tr> <tr> <td>0031</td> <td></td> <td></td> </tr> </table>																																								SYSTEMS	RADIUS	INITIAL J	0031			0031																																																											
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TITLE GRRCL: DATA FORMAT 2 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 14 PHASE \_\_\_\_\_ LABEL 84D 277 SHEET 5 OF 5

SECTION DATA CARDS

SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I/IZ \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	0.02591	0.02125	9.90E-13	12.	0.990E-9		1.0
00410000									
00410000									
IMAGINARY PART OF SCALING FACTORS									
00420000									
00420000									
00420000									
PARAMETER VALUES - UNSCALED									
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TITLE GRRCI DATA FORMAT 2 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 15 LABEL 840 277 SHEET 2 OF 4

SECTION DATA CARDS											25	35	41	49	57	65	73
SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{yz}\Delta x)_{n,n+1}$	$P_n$								
REAL PART OF SCALING FACTORS																	
00410000																	
00410000																	
00410000																	
IMAGINARY PART OF SCALING FACTORS																	
00420000																	
00420000																	
00420000																	
PARAMETER VALUES - UNSCALED																	
00430041	0002	0001	191700.	48800													
00430045		0003	27800.							1.0							
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TITLE <u>GRC1 DATA FORMAT 1</u>										PROGRAMMER _____										DATE _____									
PROBLEM NO <u>Case 15</u>										PHASE _____										SHEET <u>3</u> OF <u>4</u>									
<div style="display: flex; justify-content: space-between;"> <div>             RUN TITLE CARD              000000(62 HAA)         </div> <div>             DATA CONTROL CARD  <small>NO TYPE NO TYPE etc.</small>              0000011001300141         </div> <div>             CASE TITLE CARD              001000(62 HAA Case 15 J=87 For 1.0 to 3.5 cps)         </div> </div>																													
<div style="display: flex; justify-content: space-between;"> <div>             LOCATION CONTROL CARD              00200000         </div> <div>             EDIT CONTROL CARD              00210000         </div> <div>             GENERAL DATA CARD  <small>ω<sub>1</sub>(CPS) ω<sub>2</sub>(CPS) Δω(CPS)</small>              0030 0045 1.0 3.5 25         </div> </div>																													
<div style="display: flex; justify-content: space-between;"> <div>             SYSTEMS DATA CARDS  <small>SYSTEMS RADIUS INITIAL J</small>              0031         </div> <div>             SYSTEMS DATA CARDS  <small>SYSTEMS RADIUS INITIAL J</small>              0031         </div> </div>																													



TITLE	Corasco Case 16										PROGRAMMER											DATE																																																																																			
PROBLEM NO.	Corasco Case 16										PHASE	340										SHEET	277																																																																																		
<div style="display: flex; justify-content: space-between;"> <div> <p>RUN TITLE CARD</p> <p>000000(62 HAA SSAB(N)598 OCT. 26, 1962 Corasco Vertical Banding</p> </div> <div> <p>DATA CONTROL CARD</p> <p>NO TYPE NO TYPE etc.</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr> <td>000000</td><td>01</td><td>02</td><td>03</td><td>04</td><td>05</td><td>06</td><td>07</td><td>08</td><td>09</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td> </tr> </table> </div> </div>																																			000000	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
000000	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70																																			
<div style="display: flex; justify-content: space-between;"> <div> <p>CASE TITLE CARD</p> <p>001000(62 HAA Case 16 16 Sections + Prop. System J = 0.17 For 1.0 To 3.5 sec</p> </div> </div>																																																																																																									
<div style="display: flex; justify-content: space-between;"> <div> <p>OPTION CONTROL CARD</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr> <td>OP1</td><td>OP2</td><td>OP3</td><td>OP4</td><td>OP5</td><td>OP6</td><td>OP7</td><td>OP8</td><td>OP9</td><td>OP10</td> </tr> <tr> <td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> </table> </div> </div>																																			OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10																																																													
OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10																																																																																																
<div style="display: flex; justify-content: space-between;"> <div> <p>EDIT CONTROL CARD</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr> <td>0001</td><td>0002</td><td>0003</td><td>0004</td><td>0005</td><td>0006</td><td>0007</td><td>0008</td><td>0009</td><td>0010</td> </tr> <tr> <td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> </table> </div> </div>																																			0001	0002	0003	0004	0005	0006	0007	0008	0009	0010																																																													
0001	0002	0003	0004	0005	0006	0007	0008	0009	0010																																																																																																
<div style="display: flex; justify-content: space-between;"> <div> <p>GENERAL DATA CARD</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr> <td><math>\omega_1</math> (CPS)</td> <td><math>\omega_2</math> (CPS)</td> <td><math>\Delta\omega</math> (CPS)</td> </tr> <tr> <td>1.0</td> <td>3.5</td> <td>0.2</td> </tr> </table> </div> </div>																																			$\omega_1$ (CPS)	$\omega_2$ (CPS)	$\Delta\omega$ (CPS)	1.0	3.5	0.2																																																																	
$\omega_1$ (CPS)	$\omega_2$ (CPS)	$\Delta\omega$ (CPS)																																																																																																							
1.0	3.5	0.2																																																																																																							
<div style="display: flex; justify-content: space-between;"> <div> <p>SYSTEMS DATA CARDS</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr> <td>SYSTEMS</td> <td>RADIUS</td> <td>INITIAL J</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table> </div> </div>																																			SYSTEMS	RADIUS	INITIAL J																																																																				
SYSTEMS	RADIUS	INITIAL J																																																																																																							

TITLE GERCI DATA FORMAT 2 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 16 PHASE \_\_\_\_\_ LABEL. 8410 377 SHEET 2 OF 2

SECTION DATA CARDS

SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_n, n+1$	$(\Delta x/kAG)_{n,n+1}$	$(I_{WZ} \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	1.002591	0.002254	0.990E-12	12.	0.990E-9	1.0E+6	1.0
00410000		2	1.002591		0.				
00410000		3	1.002591		0.990E-12	1.0	0.990E-9	2591.0	1.0

IMAGINARY PART OF SCALING FACTORS

00420000		0001			0.990E-12		-0.990E-9		
00420000		0003			-0.990E-13		-0.990E-9		
00420000									

PARAMETER VALUES - UNSCALED

0043 0001	0001	0001	7500.	2.4500.	10.6	10.5	167.9		
0043 2		1	528500.	567500.	2.76	33.	81.7		
0043 3		1	1578225.	1207000.	0.624	24.	27.38		
0043 4		1	1027700.	1298000.	0.330	29.	28.95		
0043 5		1	1342000.	1575000.	0.260				
0043 7		1	1642000.	1752000.	0.236				
0043 11		1	2327950.	2266000.	0.201	30.	23.0		
0043 12		1	916000.	867000.	0.196	26.	19.5		
0043 13		1	2311400.	2850000.	0.271				
0043 18		1	241943.	524000.	0.317	9.	10.6		
0043 19		1	286600.	326000.	0.341	21.	29.12		
0043 20		1	1423660.	1686000.	0.750				
0043 27		1	661156.	808000.	1.659	22.	63.5		
0043 28		1	200224.	109300.	2.706				
0043 30		1	225770.	42800.	5.99				

TITLE GBCL DATA FORMAT 2A PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 16 PHASE \_\_\_\_\_ LABEL 

8	4	0
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 SHEET 3 OF 2

## SECTION PARAMETER VALUES - UNSCALED

[illegible]

TITLE GBRCL: DATA FORMAT 3 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. CASE 16 PHASE \_\_\_\_\_ LABEL 840 277 SHEET 4 OF 7

5	0	18	17	25	33	41	40	57	65	73
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SPECIAL CONNECTION CARDS									
n	m	SYSTEM	K <sub>n,m</sub>	C <sub>n,m</sub>	C <sub>n,m</sub> /k <sub>0</sub>	ΔX <sub>n,m</sub>	(ΔX/KAG) <sub>n,m</sub>	(I <sub>μz</sub> ΔX) <sub>n,m</sub>	
REAL PART OF SCALING FACTORS									
0051		0001	0.0	0.0		12.	0.990E-9		
0051		2	1.	1.	1.				
0051		3				1.	0.990E-9		

IMAGINARY PART OF SCALING FACTORS									
0052		0001					-0.099E-9		
0052		0003					-0.099E-9		
0052									

PARAMETER VALUES - UNSCALED									
0053	0005	0008	0001			29.	23.55		
0053	8	0011	1			28.	22.19		
0053	0013	0018	1			40	41.4		
0053	20	27	1			46	74.6		
0053	28	30	1			12	58.5		
0053	30	33	1			12	120.0		
0053	28	34	3			149.875			
0053	30	34	2	232000.					
0053	33	35	2	538000.					
0053	36		2	40.					

TITLE

PROGRAMMER

DATE

PROBLEM NO.

PHASE

LABEL

SHEET

OF

Z

5

0

13

17

25

33

41

49

57

65

73

SPECIAL CONNECTIONS - PARAMETER VALUES

	n	m	SYSTEM	k <sub>n,m</sub>	C <sub>n,m</sub>	C <sub>n,m</sub> /ω	(Δx) <sub>n,m</sub>	(Δy/Δx) <sub>n,m</sub>	(I <sub>42</sub> Δx) <sub>n,m</sub>
0053	0005	0006	0002	122 200					
0053	6		2						
0053	5	7	2	122 200					
0053	7		2						
0053	8	9	2	31 440	29.3				
0053	8	10	2	13 600	12.6				
0053	13	14	2	20 000	60.7				
0053	14	15	2	51 600	41.6				
0053	13	16	2	38 200	33.3				
0053	13	17	2	14 400	14.5				
0053	20	21	2	16 000	16.4				
0053	20	22	2	46 000	104.				
0053	20	23	2	16 700	16.7				
0053	20	24	2	80500	79.1				
0053	20	25	2	80500	79.1				
0053	20	26	2	16700	16.7				
0053	27	29	2	16800	19.8				
0053	30	31	2	1820000					
0053	30	32	2	1820000					
0053	31		2		110				
0053	32		2		110				
0053									
0053									
0053									

TITLE <u>GARCIA DATA FORMAT</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>Case 16</u>		PHASE _____		SHEET <u>4</u> OF <u>7</u>	
		LABEL <u>840</u>			

RUN TITLE CARD					
000000(62 HAA)					
DATA CONTROL CARD					
NO TYPE NO TYPE etc.					
7090101300141					
5 7 2 3 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71					
CASE TITLE CARD					
001000(62 HAA Case 16 16 SECTIONS + PROP SYSTEM S=81 For 3.5 To 22 cps					
OPTION CONTROL CARD					
OP1 OP2 OP3 OP4 OP5 OP6 OP7 OP8 OP9 OP10					
20200000					
EDIT CONTROL CARD					
00210000					
GENERAL DATA CARD					
NO. SECTIONS $\omega_1$ (CPS) $\omega_2$ (CPS) $\Delta\omega$ (CPS) $\omega = 3.5$ cps To 10.0 cps $\Delta\omega = 0.2$ cps					
0030 0036 3.5 2.5 25					
SYSTEMS DATA CARDS					
SYSTEMS RADIUS INITIAL J					
0031					
0031					



TITLE <u>GBRC-1 DATA FORMAT 1</u>		PROGRAMMER _____	DATE _____
PROBLEM NO. <u>Case 17</u>	LABEL <u>840</u>	PHASE <u>217</u>	SHEET <u>1</u> OF <u>8</u>

RUN TITLE CARD	
000000(62 HAA SSBIN) 59R	OCT 26, 1962 CONESCO VERTICAL BENDING

DATA CONTROL CARD	
NO TYPE NO TYPE etc.	
0090001 10 01 21 01 30 03 41 02 44 56 43 03 51 02 52 36 53	
5 7 2 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71	

CASE TITLE CARD	
001000(62 HAA Case 17	35 Sections + Prep System J = .17 For 1.0 to 3.5 cps

OPTION CONTROL CARD	
00200000	001 OP2 OP3 OP4 OP5 OP6 OP7 OP8 OP9 OP10
5 7 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45	

EDIT CONTROL CARD	
00210000	1 46 54 56
5 7 11 13 15 17 19 21 23 25 27 29	

GENERAL DATA CARD	
0030 0056 1.0	001(CPS) 002(CPS) Δω(CPS)
5 7 11 13 15 17 19 21 23 25 27 29	

SYSTEMS DATA CARDS	
SYSTEMS RADIUS	INITIAL J
0031	
0031	

TITLE GARCIA DATA FORMAT 2 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 17 PHASE \_\_\_\_\_ LABEL 840 277 SHEET 2 OF 8

SECTION DATA CARDS										57	65	73
SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_n, n+1$	$(\Delta x/kAG)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$P_n$			
REAL PART OF SCALING FACTORS												
00410000		0001	0.000000	0.000000	0.000000	12.	0.000000	0.	1.0			
00410000		0002	0.000000	0.	0.	1.	0.	0.	1.0			
00410000		0003	0.000000	0.	0.000000	1.	0.000000	0.000000	1.0			
IMAGINARY PART OF SCALING FACTORS												
00420000		0001	0.	0.	-0.000000			-0.000000				
00420000		0002			-0.000000			-0.000000				
00420000												
PARAMETER VALUES - UNSCALED												
00430000	0001	0001	7500.	6350.	10	4.5		114.				
0043	2	1	69700.	47800.	1.26	6.		33.5				
0043	3	1	157750.	115700.	1.38	9.		40.8				
0043	4	1	355100.	206000.	1.313	9.		21.6				
0043	5	1	242750.	227000.	0.317	12.		19.3				
0043	6	1	500475.	475000.	0.348	10		14.3				
0043	7	1	425000.	362000.	0.161	7.		6.73				
0043	8	1	341250.	355000.	0.075	7.		8.39				
0043	9	1	337000.	427000.	0.133	11.		12.5				
0043	0000	1	299450.	576000.	0.134	10.		10.0				
0043	11	1	316800.	471000.	0.0343	8.		6.42				
0043	12	1	507450.	451000.	0.073							
0043	13	1	517750.	563000.	0.0198	10		5.13				
0043	14	1	523250.	607000.	0.0224	10		8.10				
0043	17	1	534000.	585000.	0.0311							

TITLE GBRC DATA FORMAT 2A PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 17 PHASE \_\_\_\_\_ LABEL 840 277 SHEET 3 OF 8

SECTION PARAMETER VALUES - UNSCALED

SECTION NO	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{H2O})_{n,n+1}$	$P_n$
0043	0020	1	524750.	500000.	0.0706	9.	6.96		
0043	21	1	299500.	591000.	0.0679	10.	7.64		
0043	22	1	813450.	745000.	0.0857	14.	10.3		
0043	23	1	945000.	930000.	0.1070	16.	12.2		
0043	24	1	914000.	868000.	0.1000	12.	9.2		
0043	25	1	786000.	777000.	0.0994	13.	10.2		
0043	26	1	1009900.	953000.	0.1217				
0043	27	1	1515500.	1150000.	0.218				
0043	28	1	541943.	539000.	0.216	9.	10.6		
0043	29	1	286600.	336000.	0.153	5.	6.92		
0043	30	1	484100.	456000.	0.247				
0043	31	1	562000.	623000.	0.353				
0043	32	1	377500.	607000.	0.422				
0043	33	1	324500.	434000.	0.561	11.	23.2		
0043	34	1	226500.	233000.	0.619	9.	22.0		
0043	35	1	114750.	151000.	0.632	8.	23.9		
0043	36	1	101050.	91800.	0.772	5.	17.6		
0043	37	1	550000.	689000.	1.063				
0043	38	1	44200.	48600.	1.794				
0043	39	1	222770.	42800.	4.773				
0043	40	1	33420.	11200.	15-				
0043	41	2	37300.						
0043	42	2	37300.						
0043	43								



TITLE GBRC 1: DATA FORMAT 3 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 17 PHASE \_\_\_\_\_ LABEL 

8	H	0
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 SHEET 5 OF 8

SPECIAL CONNECTION CARDS									
N	M	SYSTEM	K <sub>n,m</sub>	C <sub>n,m</sub>	C <sub>n,m</sub> /ω	ΔX <sub>n,m</sub>	(ΔX/KAG) <sub>n,m</sub>	(I <sub>μz</sub> ΔX) <sub>n,m</sub>	
REAL PART OF SCALING FACTORS									
0051		0001	1.0	1.0	1.0	1.2	0.990 E-9		
0051		2	1.0	1.0	1.0	0.	0.		
0051		3	1.0	1.0	1.0	1.0	0.990 E-9		
IMAGINARY PART OF SCALING FACTORS									
0052		1					-0.99 E-9		
0052		2					-0.99 E-9		
0052									
PARAMETER VALUES - UNSCALED									
0053	0012	0013	0002	122200.					
0053	12	14	2	122200.					
0053	17	13	2	31440.	34.3				
0053	17	19	2	13600.	121.6				
0053	26	27	2	20000.	60.7				
0053	26	28	2	51600.	41.6				
0053	24	30	2	38700.	33.3				
0053	29	31	2	14400.	14.5				
0053	34	35	2	10000.	10.4				
0053	36	37	2	46000.	104.				

TITLE GBRC / DATA FORMAT 3A PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. CASE 17 LABEL - 840 277 SHEET 6 OF 8

SPECIAL CONNECTIONS - PARAMETER VALUES

	5	0	13	17	25	33	41	49	57	65	73
	N	M	SYSTEM	Kn,m	Cn,m	Cn,m/ω	(ΔX)n,m	(ΔX/KAG)n,m	(I <sub>42</sub> ΔX)n,m		
0033	0038	0037	0002	16700.	16.7						
0033	33	40	0002	16500.	79.1						
0033	38	41	0002	80000.	79.1						
0033	38	42	0002	16700.	16.7						
0033	47	48	0002	16800.	19.8						
0033	51	52	0002	183000.							
0033	51	53	0002	152000.							
0033	12	0012	0001				9.	7.36			
0033	17	20	0001				9	7.62			
0033	26	29	0001				18.	15.2			
0033	24	32	0001				22.	26.2			
0033	34	36	0001				16.	22.2			
0033	36	38	0001				15.	18.6			
0033	38	43	0001				20.	32.2			
0033	47	49	0001				7.42	30.1			
0033	49	51	0001				4.58	28.4			
0033	58	54	0001				12.	120			
0033	49	50	0002	232000							
0033	54	55	0002	538000							
0033	50	55	0002				196.5				
0033	12	14	0002		20.						
0033	14	19	0002		60.						
0033	23	23	0002		110.						
0033	32	32	0002		111.						
0033	32		0002		40						
0033	46	50					149.825				

TITLE <u>GARGI: DATA FORMAT 1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>Cass 17</u>		PHASE _____		SHEET <u>7</u> OF <u>8</u>	
LABEL <span style="border: 1px solid black; padding: 2px;">8410</span>		LABEL _____		SHEET _____	

RUN TITLE CARD		_____		_____	
000000(62 HAA		_____		_____	

DATA CONTROL CARD		_____		_____	
NO TYPE NO TYPE etc.		_____		_____	
009000100130 0141		_____		_____	
5 7 3 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71		_____		_____	

CASE TITLE CARD		_____		_____	
001000(62 HAA Cass 17 35' Sections & Prop System J = .82 For 3.5 To 25.0 cps		_____		_____	

OPTION CONTROL CARD		_____		_____	
00200000		_____		_____	
5 7 17 21 25 29 33 37 41 45		_____		_____	

EDIT CONTROL CARD		_____		_____	
00210000		_____		_____	
5 13 17 21 25 29		_____		_____	

GENERAL DATA CARD		_____		_____	
N <sub>2</sub> 3000 500 315' 17 21 25 29		_____		_____	
0030 10054 9 315' 17 21 25 29		_____		_____	
0030 10054 9 315' 17 21 25 29		_____		_____	
0030 10054 9 315' 17 21 25 29		_____		_____	

SYSTEMS DATA CARDS		_____		_____	
SYSTEMS RADIUS		_____		_____	
INITIAL J		_____		_____	
0031		_____		_____	
0031		_____		_____	



TITLE <u>GBRC 1 DATA FORMAT 1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>Case 18</u>		PHASE _____		SHEET <u>9</u> OF _____	
		LABEL <u>846</u>			

RUN TITLE CARD					
000000(62 HAA					
DATA CONTROL CARD					
NO TYPE VO TYPE etc.					
5	7	3	15	7	19
10	21	23	25	27	29
31	33	35	37	39	41
43	45	47	49	51	53
55	57	59	61	63	65
67	69	71	73	75	77
79	81	83	85	87	89
91	93	95	97	99	101

CASE TITLE CARD					
001000(62 HAA Case 18		15		Dissection r-Prop System	
				J Determined For Size Mode Shape	

OPTION CONTROL CARD					
001	002	003	004	005	006
007	008	009	010	011	012
013	014	015	016	017	018
019	020	021	022	023	024
025	026	027	028	029	030
031	032	033	034	035	036
037	038	039	040	041	042
043	044	045	046	047	048
049	050	051	052	053	054
055	056	057	058	059	060

EDIT CONTROL CARD					
001	002	003	004	005	006
007	008	009	010	011	012
013	014	015	016	017	018
019	020	021	022	023	024
025	026	027	028	029	030
031	032	033	034	035	036
037	038	039	040	041	042
043	044	045	046	047	048
049	050	051	052	053	054
055	056	057	058	059	060

GENERAL DATA CARD					
001	002	003	004	005	006
007	008	009	010	011	012
013	014	015	016	017	018
019	020	021	022	023	024
025	026	027	028	029	030
031	032	033	034	035	036
037	038	039	040	041	042
043	044	045	046	047	048
049	050	051	052	053	054
055	056	057	058	059	060

SYSTEMS DATA CARDS					
001	002	003	004	005	006
007	008	009	010	011	012
013	014	015	016	017	018
019	020	021	022	023	024
025	026	027	028	029	030
031	032	033	034	035	036
037	038	039	040	041	042
043	044	045	046	047	048
049	050	051	052	053	054
055	056	057	058	059	060









TITLE <u>GARC DATA FORMAT 1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>Case 20</u>		PHASE _____		SHEET <u>277</u> OF <u>14</u>	
LABEL <u>84C</u>					

RUN TITLE CARD _____					
000000(62 HAA _____)					

DATA CONTROL CARD _____					
NO TYPE NO TYPE etc.					
5	7	9	13	15	17
10	11	12	14	16	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72

CASE TITLE CARD _____					
001000(62 HAA Case 20 Similar To Case 19 with Special Damping - (17.045)					

OPTION CONTROL CARD _____					
00200000	001	002	003	004	005
006	007	008	009	010	011
012	013	014	015	016	017
018	019	020	021	022	023
024	025	026	027	028	029
030	031	032	033	034	035
036	037	038	039	040	041
042	043	044	045	046	047
048	049	050	051	052	053
054	055	056	057	058	059
060	061	062	063	064	065
066	067	068	069	070	071
072	073	074	075	076	077
078	079	080	081	082	083
084	085	086	087	088	089
090	091	092	093	094	095
096	097	098	099	100	101
102	103	104	105	106	107
108	109	110	111	112	113
114	115	116	117	118	119
120	121	122	123	124	125
126	127	128	129	130	131
132	133	134	135	136	137
138	139	140	141	142	143
144	145	146	147	148	149
150	151	152	153	154	155
156	157	158	159	160	161
162	163	164	165	166	167
168	169	170	171	172	173
174	175	176	177	178	179
180	181	182	183	184	185
186	187	188	189	190	191
192	193	194	195	196	197
198	199	200	201	202	203
204	205	206	207	208	209
210	211	212	213	214	215
216	217	218	219	220	221
222	223	224	225	226	227
228	229	230	231	232	233
234	235	236	237	238	239
240	241	242	243	244	245
246	247	248	249	250	251
252	253	254	255	256	257
258	259	260	261	262	263
264	265	266	267	268	269
270	271	272	273	274	275
276	277	278	279	280	281
282	283	284	285	286	287
288	289	290	291	292	293
294	295	296	297	298	299
300	301	302	303	304	305
306	307	308	309	310	311
312	313	314	315	316	317
318	319	320	321	322	323
324	325	326	327	328	329
330	331	332	333	334	335
336	337	338	339	340	341
342	343	344	345	346	347
348	349	350	351	352	353
354	355	356	357	358	359
360	361	362	363	364	365
366	367	368	369	370	371
372	373	374	375	376	377
378	379	380	381	382	383
384	385	386	387	388	389
390	391	392	393	394	395
396	397	398	399	400	401
402	403	404	405	406	407
408	409	410	411	412	413
414	415	416	417	418	419
420	421	422	423	424	425
426	427	428	429	430	431
432	433	434	435	436	437
438	439	440	441	442	443
444	445	446	447	448	449
450	451	452	453	454	455
456	457	458	459	460	461
462	463	464	465	466	467
468	469	470	471	472	473
474	475	476	477	478	479
480	481	482	483	484	485
486	487	488	489	490	491
492	493	494	495	496	497
498	499	500	501	502	503
504	505	506	507	508	509
510	511	512	513	514	515
516	517	518	519	520	521
522	523	524	525	526	527
528	529	530	531	532	533
534	535	536	537	538	539
540	541	542	543	544	545
546	547	548	549	550	551
552	553	554	555	556	557
558	559	560	561	562	563
564	565	566	567	568	569
570	571	572	573	574	575
576	577	578	579	580	581
582	583	584	585	586	587
588	589	590	591	592	593
594	595	596	597	598	599
600	601	602	603	604	605
606	607	608	609	610	611
612	613	614	615	616	617
618	619	620	621	622	623
624	625	626	627	628	629
630	631	632	633	634	635
636	637	638	639	640	641
642	643	644	645	646	647
648	649	650	651	652	653
654	655	656	657	658	659
660	661	662	663	664	665
666	667	668	669	670	671
672	673	674	675	676	677
678	679	680	681	682	683
684	685	686	687	688	689
690	691	692	693	694	695
696	697	698	699	700	701
702	703	704	705	706	707
708	709	710	711	712	713
714	715	716	717	718	719
720	721	722	723	724	725
726	727	728	729	730	731
732	733	734	735	736	737
738	739	740	741	742	743
744	745	746	747	748	749
750	751	752	753	754	755
756	757	758	759	760	761
762	763	764	765	766	767
768	769	770	771	772	773
774	775	776	777	778	779
780	781	782	783	784	785
786	787	788	789	790	791
792	793	794	795	796	797
798	799	800	801	802	803
804	805	806	807	808	809
810	811	812	813	814	815
816	817	818	819	820	821
822	823	824	825	826	827
828	829	830	831	832	833
834	835	836	837	838	839
840	841	842	843	844	845
846	847	848	849	850	851
852	853	854	855	856	857
858	859	860	861	862	863
864	865	866	867	868	869
870	871	872	873	874	875
876	877	878	879	880	881
882	883	884	885	886	887
888	889	890	891	892	893
894	895	896	897	898	899
900	901	902	903	904	905
906	907	908	909	910	911
912	913	914	915	916	917
918	919	920	921	922	923
924	925	926	927	928	929
930	931	932	933	934	935
936	937	938	939	940	941
942	943	944	945	946	947
948	949	950	951	952	953
954	955	956	957	958	959
960	961	962	963	964	965
966	967	968	969	970	971
972	973	974	975	976	977
978	979	980	981	982	983
984	985	986	987	988	989
990	991	992	993	994	995
996	997	998	999	1000	1001
1002	1003	1004	1005	1006	1007
1008	1009	1010	1011	1012	1013
1014	1015	1016	1017	1018	1019
1020	1021	1022	1023	1024	1025
1026	1027	1028	1029	1030	1031
1032	1033	1034	1035	1036	1037
1038	1039	1040	1041	1042	1043
1044	1045	1046	1047	1048	1049
1050	1051	1052	1053	1054	1055
1056	1057	1058	1059	1060	1061
1062	1063	1064	1065	1066	1067
1068	1069	1070	1071	1072	1073
1074	1075	1076	1077	1078	1079
1080	1081	1082	1083	1084	1085
1086	1087	1088	1089	1090	1091
1092	1093	1094	1095		



TITLE GBRC DATA FORMAT 3 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 20 LABEL 840 277 SHEET 14 OF \_\_\_\_\_

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{H14M})_{n,m}$
REAL PART OF SCALING FACTORS								
0051		1	1.	1.	1.	12	.9984 E-9	
0051		3	1.	1.	1.	1	.9984 E-9	
0051								

IMAGINARY PART OF SCALING FACTORS

0052		1					-.040 E-9	
0052		3					-.040 E-9	
0052								

PARAMETER VALUES - UNSCALED

0053								
0053								
0053								
0053								
0053								
0053								
0053								
0053								
0053								
0053								

TITLE	GARC: DATA FORMAT 1										PROGRAMMER	DATE									
PROBLEM NO.	Case 21										PHASE	SHEET 17 OF									
										LABEL 840 277											

RUN TITLE CARD																													
000000 (62 HAA)																													

DATA CONTROL CARD																																	
NO TYPE NO TYPE etc.																																	
5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71
00900	01	10	02	41	02	43	02	51	02	53																							

CASE TITLE CARD																													
001000 (62 HAA) Case 21 Similar to Case 19 With Special Damping - (112253T)																													

OPTION CONTROL CARD																													
021 OP2 OP3 OP4 OP5 OP6 OP7 OP8 OP9 OP10																													
020200000 3 17 21 25 29 33 37 41 45																													

EDIT CONTROL CARD																													
002100000 3 17 21 25 29																													

GENERAL DATA CARD																													
0030 5 17 25										01(CPS) 02(CPS) 03(CPS) 04(CPS)										05(CPS) 06(CPS) 07(CPS) 08(CPS)									

SYSTEMS DATA CARDS																													
SYSTEMS RADIUS										INITIAL J																			
0031																													
0031																													



TITLE GBRC DATA FORMAT 3 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 21 PHASE \_\_\_\_\_ LABEL 846 277 SHEET 19 OF \_\_\_\_\_

5	0	13	17	25	33	41	49	57	65	73
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SPECIAL CONNECTION CARDS									
n	m	SYSTEM	K <sub>n,m</sub>	C <sub>n,m</sub>	C <sub>n,m</sub> /ω	ΔX <sub>n,m</sub>	(ΔX/KAG) <sub>n,m</sub>	(I <sub>μ2</sub> ΔX) <sub>n,m</sub>	

REAL PART OF SCALING FACTORS									
0051		2001				12	.9414 E-9		
0051		0003				1	.9414 E-9		
0051									

IMAGINARY PART OF SCALING FACTORS									
0052		0001					-235 E-9		
0052		0003					-135 E-9		
0052									

PARAMETER VALUES - UNSCALED									
0053									
0053									
0053									
0053									
0053									
0053									
0053									
0053									
0053									
0053									

TITLE	GARC DATA FORMAT 1										PROGRAMMER	DATE									
PROBLEM NO.	Case 22										PHASE	SHEET 20 OF									
										LABEL	840 377										

RUN TITLE CARD																													
000000(62 HAA)																													

DATA CONTROL CARD																														
NO TYPE NO TYPE etc.																														
5	3	5	7	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71
000701	100102	41	021421	02151	02152	02153																								

CASE TITLE CARD																													
001000(62 HAA Case 22 35 Masses & Prop System increased stiffness between systems 1 and 3																													

OPTION CONTROL CARD																													
00200000	3	7	19	21	23	25	27	29	31	33	35	37	39	41	43	45													

EDIT CONTROL CARD																													
00210000	3	7	19	21	23	25	27	29																					

GENERAL DATA CARD																																							
0030										001(CPS)										002(CPS)										003(CPS)									
5										7										25																			

SYSTEMS DATA CARDS																																							
SYSTEMS										RADIUS										INITIAL J																			
0031																																							
0031																																							

TITLE GARC DATA FORMAT 2 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. Case 22 PHASE \_\_\_\_\_ LABEL 840 217 SHEET 21 OF \_\_\_\_\_

SECTION DATA CARDS

SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_n, n+1$	$(\Delta x/KAG)_{n,n+1}$	$(I_{412} \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	.002591	.002591	0.990E-12	12.	0.990E-9		1.0
00410000		3	.002591		0.990E-12	1.	0.990E-9		1.0
00410000									
IMAGINARY PART OF SCALING FACTORS									
00420000		0001			-.099E-12		-.099E-9		
00420000		3			-.099E-12		-.099E-9		
00420000									

PARAMETER VALUES - UNSCALED

0043									
0043									
0043									
0043									
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0043									

[illegible]





TITLE <u>G-BRC-1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>CASE 24 - Conesco</u>		PHASE _____		SHEET _____ OF _____	
<u>35 CONC. Masses + Propulsion Sys. UNIT MOMENT</u> <u>ABOUT HORIZ. AXIS AT PROPELLER</u>					
RUN TITLE CARD					
<u>000000(62 HAA</u>		<u>SSB(M) 578</u>		<u>Conesco Vertical Bending</u>	
DATA CONTROL CARD					
NO TYPE NO TYPE etc.					
<u>0000</u>		<u>01</u>	<u>10</u>	<u>56</u>	<u>44</u>
<u>5</u>		<u>7</u>	<u>2</u>	<u>13</u>	<u>15</u>
<u>17</u>		<u>19</u>	<u>21</u>	<u>23</u>	<u>25</u>
<u>27</u>		<u>29</u>	<u>31</u>	<u>33</u>	<u>35</u>
<u>37</u>		<u>39</u>	<u>41</u>	<u>43</u>	<u>45</u>
<u>47</u>		<u>49</u>	<u>51</u>	<u>53</u>	<u>55</u>
<u>57</u>		<u>59</u>	<u>61</u>	<u>63</u>	<u>65</u>
<u>67</u>		<u>69</u>	<u>71</u>		
CASE TITLE CARD					
<u>001000(62 HAA</u>		<u>35 SECTIONS + PROP SYSTEM</u>		<u>SINE MODE I</u>	
OPTION CONTROL CARD					
<u>00200000</u>		<u>201</u>	<u>OP2</u>	<u>OP3</u>	<u>OP4</u>
<u>5</u>		<u>17</u>	<u>21</u>	<u>25</u>	<u>29</u>
<u>33</u>		<u>37</u>	<u>41</u>	<u>45</u>	
EDIT CONTROL CARD					
<u>00210000</u>		<u>2</u>	<u>13</u>	<u>17</u>	<u>21</u>
<u>25</u>		<u>29</u>			
GENERAL DATA CARD					
<u>0030</u>		<u>5</u>	<u>9</u>	<u>17</u>	<u>25</u>
<u>33</u>		<u>37</u>	<u>41</u>	<u>45</u>	
SYSTEMS DATA CARDS					
SYSTEMS		RADIUS		INITIAL J	
<u>0031</u>					
<u>0031</u>					

TITLE G B C - 1 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
 PROBLEM NO. CASE 24 CONES LABEL 840 277 SHEET \_\_\_\_\_ OF \_\_\_\_\_

SECTION PARAMETER VALUES - UNSCALED									
SECTION NO	5	9	13	17	25	33	41	49	57
END COND.	9	13	17	25	33	41	49	57	65
SYSTEM	13	17	25	33	41	49	57	65	73
MASS	25	33	41	49	57	65	73		
WATER INERTIA	33	41	49	57	65	73			
$(\Delta x/EI)_n$	41	49	57	65	73				
$(\Delta x)_n$	49	57	65	73					
$(\Delta x/KAG)_n$	57	65	73						
$(I_{WZAK})_n$	65	73							
$P_n$	73								
0040	0001								
0041									
0042									
0043									
0044									
0045									
0046									
0047									
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0050									
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0099									
0100									

Same As Case 22, and case 16

TITLE 6800 - 1 PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_  
PROBLEM NO. CASE 24 Conneco PHASE \_\_\_\_\_ LABEL - 

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 SHEET \_\_\_\_\_ OF \_\_\_\_\_

[illegible]

TITLE <u>GABEC 1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>CASE 25 Conesco</u>		PHASE _____		SHEET _____ OF _____	
35 Conc. masses + Propulsion Sys					
Unit Vertical Force at Station 54 (STEEN)					
RUN TITLE CARD					
000000(62 HAA		SSB(N) 578		CONESCO Vertical Bending	
DATA CONTROL CARD					
NO TYPE NO TYPE etc.					
209001 10 02 43					
5 7 3 3 15 7 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71					
CASE TITLE CARD <u>CASE 26 35 masses + Prop. System Unit Vertical Force at Sta 54</u>					
001000(62 HAA					
OPTION CONTROL CARD					
CP1 CP2 CP3 CP4 CP5 CP6 CP7 CP8 CP9 CP10					
20200000 3 7 17 21 25 29 33 37 41 45					
EDIT CONTROL CARD					
00210000 3 7 17 21 25 29					
GENERAL DATA CARD					
W1(CPS) W2(CPS) W3(CPS) W4(CPS)					
0030 3 7 25					
SYSTEMS DATA CARDS					
SYSTEMS		RADIUS		INITIAL J	
0031					
0031					



TITLE <u>60EC-1</u>		PROGRAMMER _____		DATE _____	
PROBLEM NO. <u>CASE 26</u>		CONESCO		PHASE <u>3.77</u>	
35 Conc. Messes + Propulsion Sys		LABEL <u>610</u>		SHEET _____ OF _____	
UNIT Vertical Force at Station 51 (STERN)					
RUN TITLE CARD					
000000(62 HAA) <u>35(N) 578</u> <u>Conesco Vertical Banding</u>					
DATA CONTROL CARD					
NO TYPE NO TYPE etc.					
00000	01	10	58	43	
5	3	13	15	7	19
					21
					23
					25
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					37
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					67
					69
					71
CASE TITLE CARD					
001000(62 HAA) <u>35 Messes + Prop. System</u> <u>UNIT Vertical F at Station 51 (STERN)</u>					
OPTION CONTROL CARD					
00200000	01	02	03	04	05
					06
					07
					08
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GENERAL DATA CARD					
0030	01	02	03	04	05
					06
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SYSTEMS DATA CARDS					
0031	01	02	03	04	05
					06
					07
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					71



TITLE		Case 27 CONESCO		PROGRAMMER	REED	DATE	15 NOV 62
PROBLEM NO.		35 CONE MASSES + PROPELLION - SPRUNG MASSES RIGID		PHASE	LABEL	SHEET	1 OF 4
					840		

RUN TITLE CARD		NOV 15 1962		CONESCO		Vertical Brdng	
C00000(62 HAA SSAN (SRR)							

DATA CONTROL CARD		NO TYPE NO TYPE etc.	
000001	10 01 20 01 21 01 30 01 31 03 41 02 42 39 43 03 51 01 52 06 53	5 15 25 35 45 55 65 75	5 15 25 35 45 55 65 75

CASE TITLE CARD		35 MASSES + Prop System - Sprung Masses Rigid	
C00000(62 HAA Case 27			

OPTION CONTROL CARD		C01 OP2 OP3 OP4 OP5 OP6 OP7 OP8 OP9 OP10	
00200000	1 2 3 4 5 6 7 8 9 10	11 12 13 14 15 16 17 18 19 20	21 22 23 24 25 26 27 28 29 30

EDIT CONTROL CARD		C0210000	
1	31 37 39	21 25 29	33 37 41 45

GENERAL DATA CARD		C030	
000000	1.0	1.0	1.0
000000	1.0	1.0	1.0
000000	1.0	1.0	1.0

SYSTEMS DATA CARDS		INITIAL J	
SYSTEMS	RADIUS	INITIAL J	
0001	16	0001	16
0031	0001	0031	0001

TITLE \_\_\_\_\_ PROGRAMMER REED DATE 15 NOV 62  
 PROBLEM NO. Case 27 PHASE \_\_\_\_\_ LABEL 8M0 277 SHEET 2 OF 4

### SECTION DATA CARDS

SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_1 I_2 \Delta x)_{n,n+1}$	$P_n$
REAL PART OF SCALING FACTORS									
00410000		0001	002591	002591	0.990 E-12	12	0.990 E-9	2591	1.0
00410000		0002	1	1	1	1	1		1.0
00410000		0003	002591		0.990	1.0		2591	1.0
IMAGINARY PART OF SCALING FACTORS									
00420000		0001			-0.099 E-12		-0.099 E-9		
00420000		0003			-0.099 E-12				
00420000									

### PARAMETER VALUES - UNSCALED

00430001	0001	0001	7300	1350	10	4.5	114		
0043 2		1	69700	47800	126	6	53.9		
0043 3		1	157750	115700	139	9	40.8		
0043 4		1	355100	206000	1363	9	21.6		
0043 5		1	242700	367000	0.317	15	19.3		
0043 6		1	860475	425000	0.278	10	12.3		
0043 7		1	425000	365000	0.161	7	6.73		
0043 8		1	341250	325000	0.095	7	8.35		
0043 9		1	387000	427000	0.133	11	12.2		
0043 0010		1	299450	516000	0.134	10	10.0		
0043 11		1	316800	471000	0.0843	8	6.45		
0043 12		1	522050	481000	0.072	9	7.30		
0043 13		1	517750	563000	0.0798	10	8.15		
0043 14		1	583450	607000	0.0854	10	8.10		
0043 15		1	544045	582000	0.0811	9	7.65		

TITLE \_\_\_\_\_ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. Case 27 PHASE \_\_\_\_\_ LABEL 8ND SHEET 3 OF 4

SECTION PARAMETER VALUES - UNSCALED									
SECTION NO	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/ET)^m$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(\Sigma \mu z \Delta x)_{n,n+1}$	$P_n$
0043	0016	0001	5524.250	560.000	0.0106	4	6.90		
0043	17	1	579.200	591.000	0.0674	10	7.64		
0043	18	1	813.450	745.000	0.0857	14	10.80		
0043	19	1	945.000	930.000	0.1070	16	12.2		
0043	20	1	916.000	868.000	0.1000	12	9.3		
0043	21	1	706.000	777.000	0.0994	13	10.2		
0043	22	1	1040.200	953.000	0.1217	18	15.2		
0043	23	1	1531.211	1150.000	0.218	22	26.2		
0043	24	1	841.943	824.000	0.216	9	10.6		
0043	25	1	286.600	324.000	0.153	5	6.91		
0043	26	1	427.400	456.000	0.247	16	22.2		
0043	27	1	594.920	623.000	0.323	15	18.6		
0043	28	1	440.420	607.000	0.422	20	32.2		
0043	29	1	328.200	434.000	0.501	11	23.2		
0043	30	1	226.500	233.000	0.619	9	22.0		
0043	31	1	116.720	151.000	0.632	8	23.9		
0043	32	1	101.050	918.00	0.772	5	17.6		
0043	33	1	55.300	689.00	1.063	7.42	30.1		
0043	34	1	44.200	486.00	1.794	0.	0.		
0043	35	1	475.770	428.00	4.773	12.	120.		
0043	37 0001	1	35.420	1120.0	15. —				
0043	38	0003	13.600		1214.				
0043	38	3	822.0		641.5	55.125	59		
0043	39	3	2790.0		75.5	0.			1.0

TITLE \_\_\_\_\_ PROGRAMMER REED DATE 15 NOV 62  
 PROBLEM NO. Case 27 PHASE \_\_\_\_\_ LABEL 840 SHEET 4 OF 4

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	K <sub>n,m</sub>	C <sub>n,m</sub>	C <sub>n,m</sub> /ω	ΔX <sub>n,m</sub>	(ΔX/KAG) <sub>n,m</sub>	(I <sub>Hz</sub> Δx) <sub>n,m</sub>
REAL PART OF SCALING FACTORS									
0051			0001	1.0	1.0	1.0	12.0	0.990 E-9	
0051			0002	1.0	1.0	1.0			
0051			0003	1.0	1.0	1.0	1.0		

IMAGINARY PART OF SCALING FACTORS

0052			0001					-0.099 E-9	
0052									
0052									

PARAMETER VALUES - UNSCALED

0053	34	35	0002	232 000					
0053	36	38	0002	538 000					
0053	34	36	0001				4.58	20.4	
0053	37		0001		40				
0053	31	35	0002				149.875		
0053	35	38	0003				196.5		
0053									
0053									
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0053									

TITLE		PROGRAMMER		DATE																																																																																																																																																	
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<div style="display: flex; justify-content: space-between;"> <div> <p>DATA CONTROL CARD</p> <p>NO TYPE NO TYPE etc.</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr><td>0097</td><td>01</td><td>02</td><td>03</td><td>04</td><td>05</td><td>06</td><td>07</td><td>08</td><td>09</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td><td>71</td></tr> <tr><td>0097</td><td>01</td><td>02</td><td>03</td><td>04</td><td>05</td><td>06</td><td>07</td><td>08</td><td>09</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td><td>71</td></tr> </table> </div> </div>						0097	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	0097	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
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<div style="display: flex; justify-content: space-between;"> <div> <p>SYSTEMS DATA CARDS</p> <table border="1" style="width:100%; text-align: center; font-size: small;"> <tr> <td>0031</td><td>0001</td><td>16</td><td>.87</td><td>INITIAL J</td> </tr> <tr> <td>0031</td><td>0001</td><td>16</td><td>.87</td><td>INITIAL J</td> </tr> </table> </div> </div>						0031	0001	16	.87	INITIAL J	0031	0001	16	.87	INITIAL J																																																																																																																																						
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TITLE \_\_\_\_\_ PROGRAMMER REED DATE 12 NOV 62  
 PROBLEM NO. Case 30 CONESCO PHASE \_\_\_\_\_ LABEL 840 SHEET 2 OF 2  
Case 18 with Rotary Inertia Added

SECTION DATA CARDS										73
SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_n, n+1$	$(\Delta x/KAG)_{n,n+1}$	$(\Delta x \Delta x)_{n,n+1}$	$P_n$	
REAL PART OF SCALING FACTORS										
00410000		0001	0.002591	0.002591	0.990E-12	1.0	0.990E-9	2591	1.0	
00410000		0002	0.002591			1.0		2591	1.0	
00410000		0003	0.002591		.990E-12	1.0	.990E-9	2591	1.0	
IMAGINARY PART OF SCALING FACTORS										
00420000		0001			.099E-12		.099E-9			
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043 0001	0001		75.00	6350	10	4.5	114	25		
0043 2	1							87.9		
0043 3	1							408		
0043 4	1							1330		
0043 5	1							4450		
0043 6	1							4100		
0043 7	1							2195		
0043 8	1							2665		
0043 9	1							4640		
0043 10	1							4180		
0043 11	1							420		
0043 12	1									
0043 13	1							6290		
0043 14	1							9050		
0043 17	1									

TITLE \_\_\_\_\_ PROGRAMMER REED DATE 15 NOV 63  
 PROBLEM NO. \_\_\_\_\_ PHASE \_\_\_\_\_ LABEL 840 SHEET 3 OF 2

SECTION PARAMETER VALUES - UNSCALED									
5	9	13	17	25	33	41	49	57	65
SECTION NO	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{H2O})_{n,n+1}$	$P_n$
0043	0020	0001						7930	
0043	21	1						10000	
0043	22	1						14120	
0043	23	1						16000	
0043	24	1						12350	
0043	25	1						11290	
0043	26	1							
0043	29	1						3060	
0043	32	1						1278	
0043	33	1							
0043	34	1							
0043	36	1							
0043	38	1							
0043	43	1						1050	
0043	44	1						663	
0043	45	1						390	
0043	46	1						378	
0043	47	1							
0043	49	1							
0043	51	1							
0043	540002	1							
0043	13	2							
0043	14	2							
0043									

TITLE \_\_\_\_\_ PROGRAMMER RED DATE 15 NOV 62  
 \_\_\_\_\_ PHASE \_\_\_\_\_ LABEL 

8	4	0
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 SHEET 277 OF 2  
 PROBLEM NO. \_\_\_\_\_

**SECTION PARAMETER VALUES - UNSCALED**

[illegible]

TITLE \_\_\_\_\_ PROGRAMMER REF DATE 15 NOV 62  
 PROBLEM NO. \_\_\_\_\_ PHASE \_\_\_\_\_ LABEL 8HD 277 SHEET 5 OF 7

5	0	18	17	25	33	41	40	57	63	73
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SPECIAL CONNECTION CARDS						
$n$	$m$	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$
REAL PART OF SCALING FACTORS						
0051		0001	1.0	1.0	1.0	1.2
0051		2	1.0	1.0	1.0	0.
0051		3	1.0	1.0	1.0	1.

IMAGINARY PART OF SCALING FACTORS						
0052		0001				-0.099 E-9
0052		0003				-0.099 E-9
0052						

PARAMETER VALUES - UNSCALED						
0033	0012	0013	0002			
0033	12	14	2			
0033	17	18	2			
0033	17	19	2			
0033	21	22	2			
0033	26	28	2			
0033	29	30	2			
0033	29	31	2			
0033	34	35	2			
0033	36	37	2			

**PROBLEM NO.** \_\_\_\_\_

[illegible]

TITLE \_\_\_\_\_ PROGRAMMER REED DATE 15 NOV 62  
 PROBLEM NO. \_\_\_\_\_ PHASE \_\_\_\_\_ LABEL - 

8	4	0
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 SHEET 27 OF 7

**SPECIAL CONNECTIONS - PARAMETER VALUES**

[illegible]

**CONESCO, INC., consultants in engineering science**

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Theory of Elasticity  
McGraw-Hill (1951)

<p>CONESCO, Inc., Cambridge, Mass., Rep. F-111-2</p> <p>SHIP HULL VIBRATIONS-5, Analysis of Hull Structures as applied to SS8(N) 598 GEORGE WASHINGTON by F. E. Reed and E. Cuthill, Mar. 1963, XI 270 p. graphs, tables, refs. (Index No. 5-F013-11-01, Task 1351; Contract NOs 86111)</p> <p>A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SS8(N) 598 GEO. WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stem of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.</p>	<ol style="list-style-type: none"> <li>1. Submarines (Nuclear)- Vibration-Mathematical Analysis</li> <li>2. Propellers (Marine)- Vibration</li> <li>3. Vibration - Analysis GEORGE WASHINGTON</li> <li>4. TON (U. S. fleet ballistic-missile submarine, nuclear, SS8(N) 598)</li> </ol> <p>I Reed, F. E. II Cuthill, Elizabeth</p>
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